These answers would earn full credit. Other methods also may be valid.

(8) 1. Calculate the derivative of y with respect to x if $\sin(x+y) = x + \cos(y)$.
Answer We $\frac{d}{dx}$ the equation $\sin(x+y) = x+\cos(y)$. The result is $\cos(x+y)(1+y') = 1 - \sin(y)y'$. Now solve for y': $\cos(x+y)y' + \sin(y)y' = 1 - \cos(x+y)$ and $y' = \frac{1-\cos(x+y)}{\cos(x+y)+\sin(y)}$.

Section 3.8, exercise 23

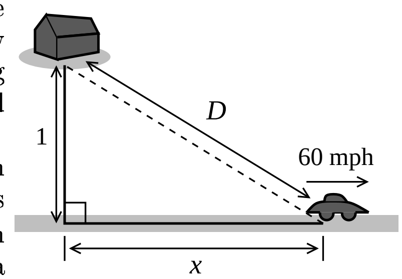
(10) 2. a) Calculate the derivative if $y = \arctan\left(\frac{1+t}{1-t}\right)$. **Answer** $y' = \frac{1}{1+\left(\frac{1+t}{1-t}\right)^2}\left(\frac{1(1-t)-(-1)(1+t)}{(1-t)^2}\right)$.

The textbook indicates this simplifies to $\frac{1}{1+t^2}$. This is *not needed here.* Section 3.9, exercise 31

b) Find an equation of the tangent line at the point indicated: $f(x) = \ln(x^2)$, $x = 4$.
Answer If $f(x) = \ln(x^2)$, then $f'(x) = \left(\frac{1}{x^2}\right)2x = \frac{2}{x}$. Then $f(4) = \ln(16)$ and $f'(4) = \frac{1}{2}$, so an equation for the tangent line is $y - \ln(16) = \frac{1}{2}(x-4)$.

Section 3.10, exercise 30

(12) 3. A road perpendicular to a highway leads to a farmhouse located 1 mile away. An automobile travels past the farmhouse at a speed of 60 mph. How fast is the distance between the automobile and the farmhouse increasing when the automobile is 3 miles past the intersection of the farmhouse and the road?



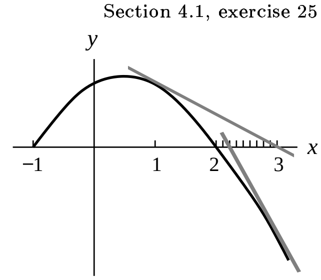
Answer Suppose x is the distance the automobile is from the point on the highway which is closest to the house. We know $\frac{dx}{dt} = 60$. If D is the distance from the automobile to the house, we want to know $\frac{dD}{dt}$ when $x = 3$. The point on the road, the house, and the car are the vertices of a right triangle, so $D^2 = x^2 + 1^2$. We differentiate and get $2D\frac{dD}{dt} = 2x\frac{dx}{dt}$ so $\frac{dD}{dt} = \left(\frac{x}{D}\right)\frac{dx}{dt}$. When $x = 3$, $D = \sqrt{10}$ so that when the automobile is 3 miles past the intersection, $\frac{dD}{dt} = \left(\frac{3}{\sqrt{10}}\right)60$.

Section 3.11, exercise 9

(6) 4. The cube root of 27 is 3. How much larger is the cube root of 27.2? Estimate using the Linear Approximation.
Answer If $f(x) = x^{1/3}$, then $f'(x) = \frac{1}{3}x^{-2/3}$. Certainly $f(27) = 3$ and $f'(27) = \frac{1}{3}(27^{-2/3}) = \frac{1}{27}$. The Linear Approximation to 27.2 is $f(27) + f'(27)(.2) = 3 + \frac{.2}{27}$. The cube root of 27.2 is approximately $\frac{.2}{27}$ larger than 3.

Section 4.1, exercise 25

(8) 5. Let x_1, x_2 be the estimates obtained by applying Newton's Method with $x_0 = 1$ to the function graphed in the accompanying figure. Estimate the numerical values of x_1 and x_2 and draw the tangent lines used to obtain them.
Answer $x_1 \approx 3.0$, $x_2 \approx 2.2$



Section 4.8, exercise 19

(10) 6. Find the maximum and minimum values of the function on the given interval. $y = x - \frac{4x}{x+1}$, $[0,3]$
Answer Max and min values occur at endpoints or critical points. y's value at 0 is 0 and y's value at 3 is $3 - \frac{12}{4} = 0$. Now for the critical points: $y' = 1 - \frac{4(x+1)-1(4x)}{(x+1)^2} = 1 - \frac{4}{(x+1)^2}$. This is 0 if $1 = \frac{4}{(x+1)^2}$ or $(x+1)^2 = 4$ or $x+1 = \pm 2$ or $x = -1 \pm 2$. -3 is not in the domain, so the only relevant critical point is at $x = 1$ where $y = 1 - \frac{4}{2} = -1$. The maximum value is 0 and the minimum value is -1. Section 4.2, exercise 39

(9) 7. Find the critical points and the intervals on which the function is increasing or decreasing, and apply the First Derivative Test to each critical point. $y = \cos\theta + \sin\theta$, $[0, 2\pi]$
Answer If $y = \cos\theta + \sin\theta$, then $y' = -\sin\theta + \cos\theta$. The critical points occur when $-\sin\theta + \cos\theta = 0$ and this is when $\tan\theta = 1$. One such θ is $\frac{\pi}{4}$ and another (tangent is periodic with period π) is $\frac{5\pi}{4}$. Now $y' = 1$ at 0, $y' = -1$ at π, and certainly $y' = 1$ at 2π. Therefore continuity of y' implies that $y' > 0$ to the left of $\frac{\pi}{4}$ and $y' < 0$ to the right of $\frac{\pi}{4}$, so y has a local maximum at $\frac{\pi}{4}$. Since $y' < 0$ to the left of $\frac{5\pi}{4}$ and $y' > 0$ to the right of $\frac{5\pi}{4}$, y has a local minimum at $\frac{5\pi}{4}$. y is increasing in $[0, \frac{\pi}{4}]$ and $[\frac{5\pi}{4}, 2\pi]$. It is decreasing in $[\frac{\pi}{4}, \frac{5\pi}{4}]$.

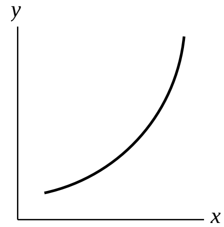
Section 4.3, exercise 42

(11) 8. Determine the intervals on which the function is concave up or down and find the points of inflection. $y = (x^2 - 3)e^x$

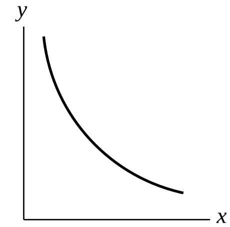
Answer If $y = (x^2-3)e^x$ then $y' = 2xe^x + (x^2-3)e^x = (x^2+2x-3)e^x$ and $y'' = (2x+2)e^x + (x^2+2x-3)e^x = (x^2+4x-1)e^x$. Since $e^x > 0$ always, the sign and "zero" information is controlled by x^2+4x-1. The roots of this quadratic are $\frac{-4\pm\sqrt{16+4}}{2} = -2\pm\sqrt{5}$. At 0, the quadratic is -1. Therefore the function has inflection points at $x = -2\pm\sqrt{5}$. It is concave down in $(-2-\sqrt{5}, -2+\sqrt{5})$ and is concave up in both $(-\infty, -2-\sqrt{5})$ and $(-2+\sqrt{5}, \infty)$.

Section 4.4, exercise 17

(4) 9. Sketch an arc where f' and f'' have the sign combination $++$ on axes (A). Do the same for $-+$ on axes (B).



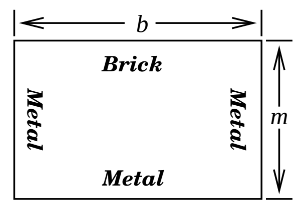
(A)



(B)

Section 4.4, preliminary question 1

(12) 10. A landscape architect wishes to enclose a rectangular garden on one side by a brick wall costing \$30/ft and on the other sides by a metal fence costing \$10/ft. If the area of the garden is 1,000 ft^2, find the dimensions of the garden that minimizes the cost.



Answer Suppose the brick wall is b feet long and the other side of the rectangle is m feet long. Then the area of the rectangle is bm and the cost of the enclosure is $30b + 10b + 20m = 40b + 20m$. We know that $bm = 1{,}000$ so $b = \frac{1{,}000}{m}$ and therefore the cost is $C(m) = \frac{40{,}000}{m} + 20m$. The domain for this problem is all m's in $(0,\infty)$. Certainly $\lim_{m\to 0^+} C(m) = \infty$ because of the first term in $C(m)$, and $\lim_{m\to\infty} C(m) = \infty$ because of the second term. Therefore a minimum will occur "inside" the interval at a critical point. Now $C'(m) = -\frac{40{,}000}{m^2} + 20$ which is 0 when $m^2 = 2{,}000$ or when $m = 20\sqrt{5}$ (the only critical point in the domain). Then the constraint equation gives $b = \frac{50}{\sqrt{5}}$ and these are the dimensions of the garden which minimizes cost.

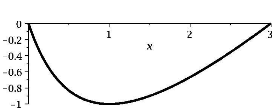
Section 4.6, exercise 11

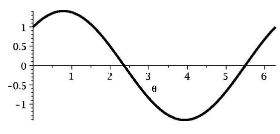
(10) 11. Evaluate the limit. Be sure, as the cover page states, to **Show your work** since **An answer alone may not receive full credit.** Explain why any special method you use is applicable. $\lim_{x\to 4} \frac{1}{\sqrt{x}-2} - \frac{4}{x-4}$

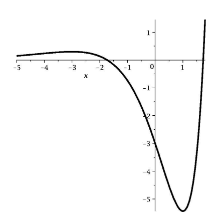
Answer First, do some algebra to get one simple fraction: $\frac{1}{\sqrt{x}-2} - \frac{4}{x-4} = \frac{(x-4)-4(\sqrt{x}-2)}{(\sqrt{x}-2)(x-4)} = \frac{(x-4)-4\sqrt{x}+8}{(\sqrt{x}-2)(x-4)} = \frac{x-4\sqrt{x}+4}{(\sqrt{x}-2)(x-4)}$. When $x = 4$, the top is $4 - 4\sqrt{4} + 4 = 8 - 4\cdot 2 = 0$ and the bottom is 0 also, so we can try L'H. The derivative of the top is $1 - \frac{2}{\sqrt{x}}$ and the derivative of the bottom is $\left(\frac{1}{2\sqrt{x}}\right)(x-4) + (\sqrt{x}-2)1$. Therefore we should compute $\lim_{x\to 4} \frac{1-\frac{2}{\sqrt{x}}}{\left(\frac{1}{2\sqrt{x}}\right)(x-4)+(\sqrt{x}-2)}$. When $x = 4$ the top is $1 - \frac{2}{\sqrt{4}} = 1 - \frac{2}{2} = 0$ and the bottom is $\left(\frac{1}{2\sqrt{4}}\right)(4-4) + (\sqrt{4}-2) = 0$, so we can try L'H. The derivative of the top is $x^{-3/2}$ and the derivative of the bottom is $-x^{-3/2}(x-4) + \left(\frac{1}{2\sqrt{x}}\right)1 + \frac{1}{\sqrt{x}}$. So we compute $\lim_{x\to 4} \frac{x^{-3/2}}{-x^{-3/2}(x-4)+\left(\frac{1}{2\sqrt{x}}\right)1+\frac{1}{2\sqrt{x}}}$. Here when we "plug in" $x = 4$ (officially, we use continuity) the result is $\frac{4^{-3/2}}{-4^{-3/2}(4-4)+\left(\frac{1}{2\sqrt{4}}\right)1+\frac{1}{2\sqrt{4}}}$ which is $\frac{\frac{1}{8}}{\frac{1}{4}+\frac{1}{4}}$. This answer is fine since the bottom is $\neq 0$, but it also "simplifies" to $\frac{1}{4}$ which is *not needed here.*

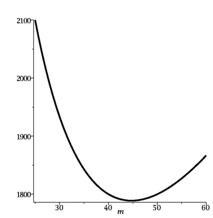
Section 4.7, exercise 28

Comment We asked students to solve almost all of these problems and hand in their solutions. Many should be familiar. Here are some graphs for a few problems.


The function in #6


The function in #7


The function in #8


The function in #10