

1. Find the limits for the following indeterminate forms of the type “ $\infty - \infty$ ”.

a)  $\lim_{x \rightarrow 0} \frac{1}{\sin x} - \frac{1}{x}$ .

b)  $\lim_{x \rightarrow 0} \frac{1}{x^2} - \frac{1}{x}$ .

c)  $\lim_{x \rightarrow 0} \frac{1+x}{x} - \frac{1-x}{x}$ .

2. Questions about asymptotic growth “near”  $\infty$  occur naturally when computer scientists analyze algorithms. One seemingly simple problem is sorting. How many comparisons are required to sort a list of  $n$  numbers? *Sorting and Searching*, volume 3 of *The Art of Computer Programming*, by D. Knuth, gives the following average running times for several sorting algorithms as a function of  $n$ :

Name	Running time
Comparison	$4n^2 + 10n$
Merge exchange	$3.7n(\ln n)^2$
Heapsort	$23.08n \ln n + 0.2n$

Which sorting method would you rather use if, in your application,  $10 \leq n \leq 20$  (e.g., sorting a bridge hand)? Which would you rather use if  $100 \leq n \leq 150$  (e.g., sorting grades in a lecture course)? Which would you rather use if  $n \approx 10^6$  (e.g., sorting license plate numbers in New Jersey)? What happens to these functions as  $n \rightarrow \infty$ ?

3. A charged particle moves along the  $x$ -axis under the influence of an electric field. The field strength varies with time, and as a result the velocity of the particle is complicated. The position of the particle at time  $t$  is written as  $x = x(t)$  and the velocity of the particle at time  $t$  is written as  $v = v(t)$ .

Suppose we know that  $x(0) = 0$ , and also that

$$v(t) = \begin{cases} 2t - 1, & \text{if } 0 \leq t \leq 1 \\ 4t - 3, & \text{if } 1 \leq t \leq 2 \\ 6t - 7, & \text{if } 2 \leq t \leq 3 \end{cases}.$$

What is  $x(1)$ ? What is  $x(2)$ ? What is  $x(3)$ ? Sketch the graphs of  $x = x(t)$  and  $v = v(t)$ .

4. a) A car is traveling at 50 mi/h when the brakes are fully applied, producing a constant deceleration of 40 ft/s<sup>2</sup>. What is the distance covered before the car comes to a stop?

b) A car braked with a constant deceleration of 40 ft/s<sup>2</sup> and produced skid marks measuring 160 feet before coming to a stop. How fast was the car traveling when the brakes were first applied?

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One problem will be selected for a writeup to be handed in at the next recitation meeting. Please see Professor Greenfield’s Math 153 webpage to learn which problem to hand in.