

1. Let a be a positive constant and consider the functions

$$f(x) = \arcsin\left(\frac{x}{a}\right) \quad \text{and} \quad g(x) = a \arctan\left(\frac{x}{a}\right).$$

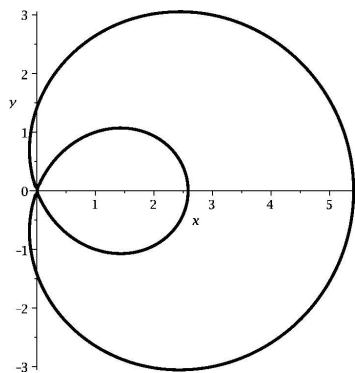
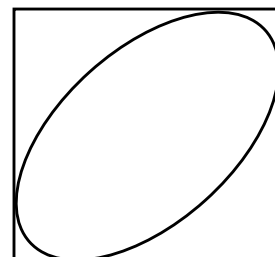
Find the derivatives of f and g and express them in as simple a form as possible. There is a certain value of a for which the lines tangent to the graphs of these two functions at $x = 1$ are parallel lines. Find that value of a to 3-place accuracy. (Find an exact equation satisfied by a , and then get an accurate enough solution from your calculator.)

2. Suppose $Z(x) = \frac{3x - 5}{2x - 1}$.

- Compute $Z'(x)$. What are $Z(1)$ and $Z'(1)$?
- Suppose s is any number in the domain of Z , and $t = Z(s)$. Solve this equation for s . Explain why solving this equation verifies that Z is one-to-one.
- Find W , the function inverse to Z , and compute W' . What are $W(-2)$ and $W'(-2)$?
- Explain the relationships between the numerical answers in a) and c).

3. The graph of the tilted ellipse $x^2 - xy + y^2 = 3$ is shown to the right. What are the dimensions and the location of the box containing the ellipse?

Note The sides of the box are vertical and horizontal and also are tangent to the ellipse.



4. Find an equation of the tangent line at the four points on the curve $(x^2 + y^2 - 4x)^2 = 2(x^2 + y^2)$, where $x = 1$. This curve, shown to the left, is an example of a *limaçon of Pascal* named after the father of the French philosopher Blaise Pascal, who first described it in 1650.

This is essentially problem 50 in section 3.8 of the textbook.

One problem will be selected for a writeup to be handed in at the next recitation meeting. Please see Professor Greenfield's Math 153 webpage to learn which problem to hand in.