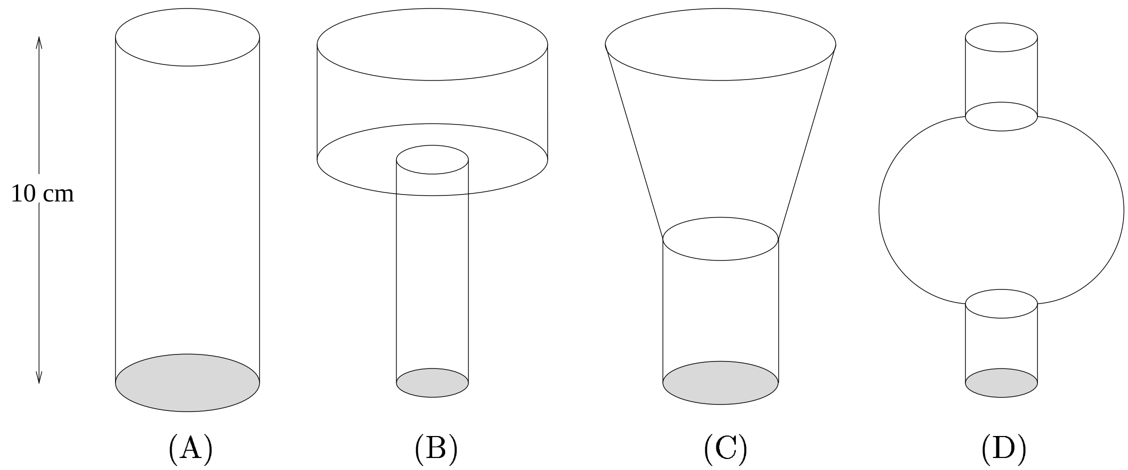
1. Four containers are each 10 cm tall. Each of them has a volume of 30 cm^3 and each is being filled by a liquid at the rate of 5 cm^3 per minute. Here is a picture of the containers:



a) For each of the containers, graph the height, $h(t)$, of the level of the liquid in the containers measured in centimeters as a function of time, t, measured in minutes.

b) Which of the functions graphed in a) are continuous? Explain your answers.

c) Which of the functions graphed in a) are differentiable? Explain your answers.

2. For any constant c, define the function f_c with the formula $f_c(x) = x^3 + 2x^2 + cx$.

a) Graph $y = f_c(x)$ for these values of the parameter c: $c = -1, 0, 1, 2, 3, 4$. What are the similarities and differences among the graphs, and how do the graphs change as the parameter increases?

b) For what values of the parameter c will f_c have one local maximum and one local minimum? Use calculus. As c increases, what happens to the distance between the local maximum and the local minimum?

c) For what values of the parameter c will f_c have no local maximum or local minimum? Use calculus.

d) Are there any values of the parameter c for which f_c will have exactly one horizontal tangent line?

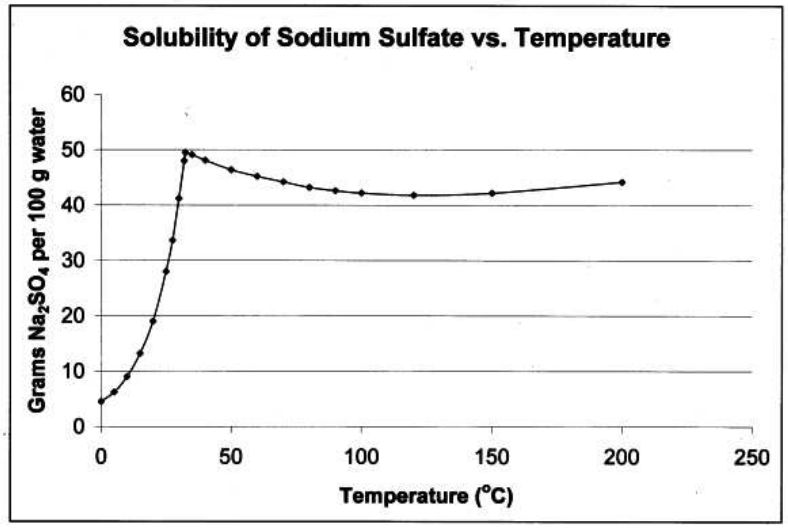
3. a) Suppose you know that $f'(x) = (x-1)(x-2)^2(x-3)^3(x-4)^4(x-5)^5$. What are the critical points of f? Which of them are local extrema, and what kind of local extrema are they?

b) Suppose you know that $g'(x) = x(x-1)^{2/3}(x-2)^{3/5}(x-3)^{4/7}$. What are the critical points of g? Which of them are local extrema, and what kind of local extrema are they?

Note You are *not* asked to compute f and g explicitly.

OVER

4. The amount of a substance which can be dissolved in a solution may vary with temperature. Below is a graph of the solubility (the maximum amount of the substance) in grams of sodium sulfate, Na_2SO_4, which can be dissolved in 100 grams of water as a function of temperature in degrees Celsius. Suppose $S(T)$ is the solubility at temperature T. Use the graph to answer the following questions as well as you can.



a) Where is $S(T)$ continuous? Where is $S(T)$ differentiable?

b) Where is $S(T)$ increasing? Where is it decreasing? Does $S(T)$ have any local extrema? If yes, where and what type?

c) In what intervals is $S(T)$ concave up? In what intervals is $S(T)$ concave down? Does $S(T)$ have any points of inflection?

d) Sketch a graph of $S'(T)$. What are the units on each axis of your graph?

5. Suppose you know that $f'(x) = \dfrac{2}{1+x^4} - \dfrac{3}{4+x^4}$. Is $f(0) < f(1)$?

Note You probably can't write a formula for a function with this derivative at this time. Here is such a function:

$$f(x) = \frac{\sqrt{2}}{4}\ln\left(\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}\right) + \frac{\sqrt{2}}{2}\arctan(\sqrt{2}x+1) + \frac{\sqrt{2}}{2}\arctan(\sqrt{2}x-1)$$

$$+\frac{3}{16}\ln(x^2-2x+2) - \frac{3}{8}\arctan(x-1) - \frac{3}{16}\ln(x^2+2x+2) - \frac{3}{8}\arctan(x+1)\,.$$

Does knowing this formula help or is studying the derivative easier? Please make an *indirect* argument, using information about f'.

One problem will be selected for a writeup to be handed in at the next recitation meeting. Please see Professor Greenfield's Math 153 webpage to learn which problem to hand in.