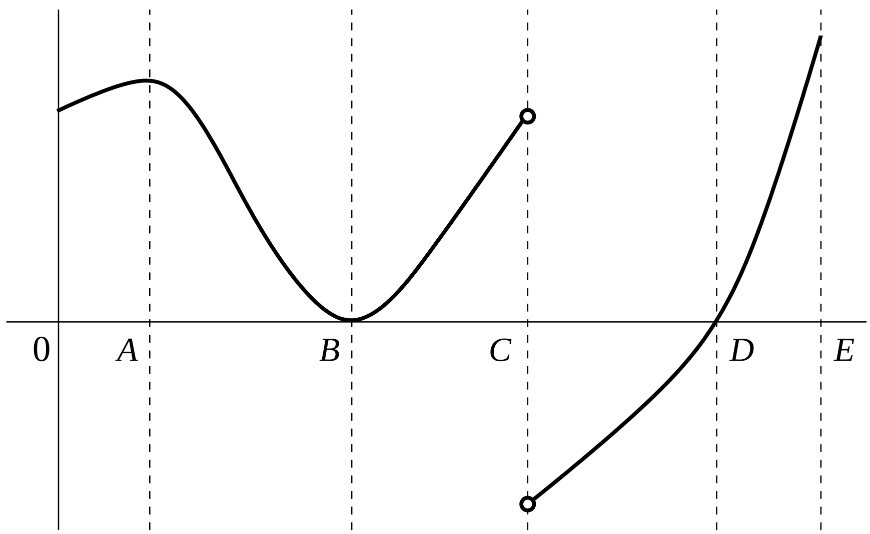
1. Suppose $5x^3y - 3xy^2 + y^3 = 6$. $(1,2)$ is a point on this curve. Is the curve concave up or concave down at $(1,2)$?

Explicit way to go y *can* be solved as a function of x.* Then you can differentiate the formula twice and evaluate when $x = 1$.

Implicit way to go Find $\frac{dy}{dx}$ implicitly and then differentiate again to get $\frac{d^2y}{dx^2}$. Evaluate everything at $(1,2)$.

2. Suppose that $y = f(x)$ is a continuous function defined on the interval from $x = 0$ to $x = E$. Below is a graph of $f'(x)$, the derivative of $f(x)$, which is defined at all points of $[0, E]$ except at $x = C$.



A graph of $f'(x)$, where it is defined

a) Where is $f(x)$ increasing? Where is $f(x)$ decreasing? Where does $f(x)$ have local extreme values (for $0 < x < E$)?

b) Where is $f(x)$ concave up? Where is $f(x)$ concave down? Where does $f(x)$ have inflection points?

c) Draw a possible graph of $f(x)$ which uses all information given and deduced about $f(x)$.

OVER

* Here it is (really!):

$$y = \left(-\frac{5}{2}x^4 + 3 + x^3 + \frac{1}{18}\sqrt{1500x^9 - 675x^8 - 4860x^4 + 2916 + 1944x^3}\right)^{1/3}$$

$$- \frac{\frac{5}{3}x^3 - x^2}{\left(-\frac{5}{2}x^4 + 3 + x^3 + \frac{1}{18}\sqrt{1500x^9 - 675x^8 - 4860x^4 + 2916 + 1944x^3}\right)^{1/3}} + x$$

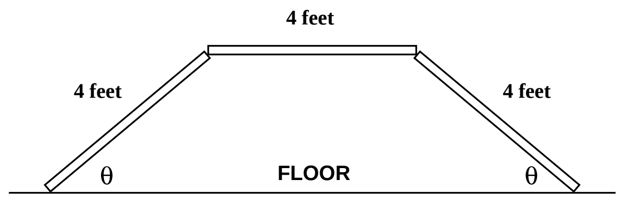
3. Consider the functions given by the equation $f_c(x) = (x^2 + c)e^x$, where c is a parameter.

a) Use the calculator to observe the curves for the values $c = 0, 1, 2$ when x is in the interval $[-4, 1]$. Do all three curves have the same number of horizontal tangents? Do all three curves have the same number of inflection points? You may have to *zoom* in to investigate this.

b) Use calculus to determine the location of all the inflection points of the graph of $y = f_c(x)$. Your answer may depend on c.

c) At what values of c does the number of inflection points change? What are the values of c (if any) for which there is exactly one inflection point?

4. A child wants to build a tunnel using three equal boards, each 4 feet wide, one for the top and one for each side as shown. The two sides are to be tilted at equal angles θ to the floor. What is the maximum cross-sectional area A that can be achieved?



One problem will be selected for a writeup to be handed in at the next recitation meeting. Please see Professor Greenfield's Math 153 webpage to learn which problem to hand in.