1A

(16) 1. a) Find parametric equations for the line L containing $M = (1, -2, 3)$ and $N = (4, 0, 2)$.

Answer $\begin{cases} x = \_\_\_\_\_\_ \\ y = \_\_\_\_\_\_ \\ z = \_\_\_\_\_\_ \end{cases}$

b) Find an equation for the plane P through $A = (0, 0, 1)$, $B = (2, 0, -1)$, and $C = (3, 3, 0)$.

An equation for the plane is ________________.

c) The line L and the plane P intersect. Find coordinates for the point of intersection.

The point of intersection is $(\quad , \quad , \quad)$.

(14) 2. Suppose the position vector of a curve is given by $\mathbf{r}(t) = \langle e^{-2t}, e^t, t^2 \rangle$. Find the unit tangent vector and the unit normal vector to this curve when $t = 0$. That is, find $\mathbf{T}(0)$ and $\mathbf{N}(0)$.

$\mathbf{T}(0) = \langle \quad , \quad , \quad \rangle$ $\qquad$ $\mathbf{N}(0) = \langle \quad , \quad , \quad \rangle$

(14) 3. Suppose $f(x, y) = x + y^2$.

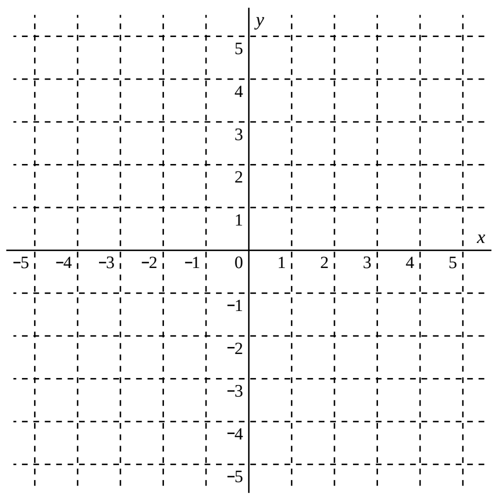
a) Compute $\nabla f(x, y)$.

$\nabla f(x, y) = \langle \quad , \quad \rangle$

b) Graph the level curves of f which pass through the points $P = (1, 1)$ and $Q = (1, -2)$ on the axes given.

c) Compute $\nabla f(1, 1)$ and $\nabla f(1, -2)$. On the same axes as your answer to b), as well as you can, sketch $\nabla f(1, 1)$ <u>starting at P</u> and $\nabla f(1, -2)$ <u>starting at Q</u>.

$\nabla f(1, 1) = \langle \quad , \quad \rangle$ and $\nabla f(1, -2) = \langle \quad , \quad \rangle$



(16) 4. Suppose $f(t)$ is a differentiable function of one variable, and $f(1) = A$, $f'(1) = B$, and $f''(1) = C$. Define $F(x, y, z)$ using this equation: $F(x, y, z) = f(xz^2 - y^3)$.

a) Compute $F(1, 2, 3)$ in terms of the information supplied and any needed constants.

$F(1, 2, 3) = \_\_\_\_$

b) Compute $\frac{\partial F}{\partial x}(1, 2, 3)$ in terms of the information supplied and any needed constants.

$\frac{\partial F}{\partial x}(1, 2, 3) = \_\_\_\_$

c) Compute $\frac{\partial F}{\partial z}(1, 2, 3)$ in terms of the information supplied and any needed constants.

$\frac{\partial F}{\partial z}(1, 2, 3) = \_\_\_\_$

d) Compute $\frac{\partial^2 F}{\partial z^2}(1, 2, 3)$ in terms of the information supplied and any needed constants.

$\frac{\partial^2 F}{\partial z^2}(1, 2, 3) = \_\_\_\_$

e) Compute $\frac{\partial^2 F}{\partial x \partial z}(1, 2, 3)$ in terms of the information supplied and any needed constants.

$\frac{\partial^2 F}{\partial x \partial z}(1, 2, 3) = \_\_\_\_$

(8) 5. Suppose z is implicitly defined as a function of x and y by the equation $x^2y - 7xz^3 + zy = 5$. Find a formula for $\frac{\partial z}{\partial x}$.

$\frac{\partial z}{\partial x} =$ ____.

(16) 6. In this problem, the function f is defined by $f(x, y, z) = \sin(2xy)z + yz^2$.

a) Compute $\nabla f(x, y, z)$ and find its value at $p = (0, 2, -1)$.

$\nabla f(x, y, z) = \langle \quad , \quad , \quad \rangle$

$\nabla f(0, 2, -1) = \langle \quad , \quad , \quad \rangle$

b) Use your answer to a) to find

i) The value of the largest directional derivative of f at p and its direction (unit vector!).

The value of the largest directional derivative is ____.

The direction of the largest directional derivative is $\langle \quad , \quad , \quad \rangle$.

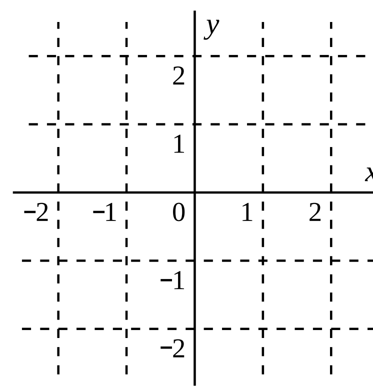
ii) The directional derivative for f at p in the direction from p to $q = (2, 2, 3)$.

The value of this directional derivative is ____.

iii) One unit vector $\vec{u}$ so that the directional derivative of f at p in the direction of u is 0. (The magnitude of $\vec{u}$ should be 1!)

The direction of a vector with directional derivative equal to 0 is $\langle \quad , \quad , \quad \rangle$.

(16) 7. An ellipse in $\mathbb{R}^2$ has parametric equations $\begin{cases} x = 2\cos t \\ y = \sin t \end{cases}$.



a) Sketch this ellipse on the axes to the right.

b) Write an integral expression for the total length of the ellipse. Do *not* compute the value of this expression.

The total length of the ellipse is ____.

c) For such a curve, the curvature, κ, can be computed by $\kappa = \frac{|y''(t)x'(t) - x''(t)y'(t)|}{(x'(t)^2 + y'(t)^2)^{3/2}}$. Compute κ for this ellipse. Use this formula to find the curvature at the point $P = (2, 0)$ and at the point $Q = (0, -1)$.

A formula for κ is ____. At $P = (2, 0)$, $\kappa =$ ____. At $Q = (0, -1)$, $\kappa =$ ____.

A A

First Exam for Math 251, sections 1, 2 & 3

February 25, 2010

NAME ____

Do all problems, in any order.

Show your work. An answer alone may not receive full credit.

No notes and no calculators may be used on this exam.

"Simplification" of answers is not necessary,
but standard values of traditional functions
such as e^0 and $\sin\left(\frac{\pi}{2}\right)$ should be given.

Problem Number	Possible Points	Points Earned:
1	16	
2	14	
3	14	
4	16	
5	8	
6	16	
7	16	
Total Points Earned:		

A A