(20) 1. a) Find parametric equations for the line L containing $M = (1, -2, 3)$ and $N = (4, 0, 2)$.

Answer $\begin{cases} x = \_\_\_\_\_\_\_\_ \\ y = \_\_\_\_\_\_\_\_ \\ z = \_\_\_\_\_\_\_\_ \end{cases}$

b) Find an equation for the plane P through $A = (0, 0, 1)$, $B = (2, 0, -1)$, and $C = (3, 3, 0)$.

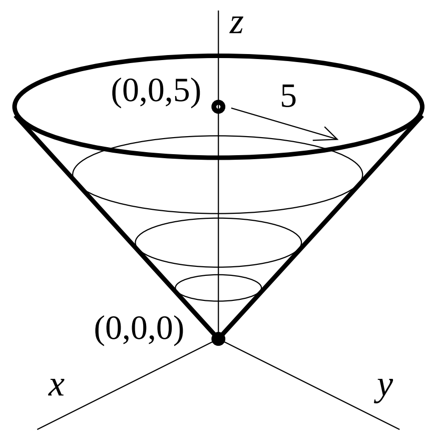
An equation for the plane is ________________________**.**

c) The line L and the plane P intersect. Find coordinates for the point of intersection.

The point of intersection is $\left(\quad , \quad , \quad \right)$**.**

d) Are L and P perpendicular? Briefly explain the reason for your answer.

(20) 2. Suppose C is the solid right circular cone shown to the right. Its height is 5 units, with vertex at the origin, $(0, 0, 0)$, and the base is centered at $(0, 0, 5)$ with radius equal to 5. Compute the triple integral of $\frac{z}{(x^2+y^2+1)^3}$ over the solid object C.

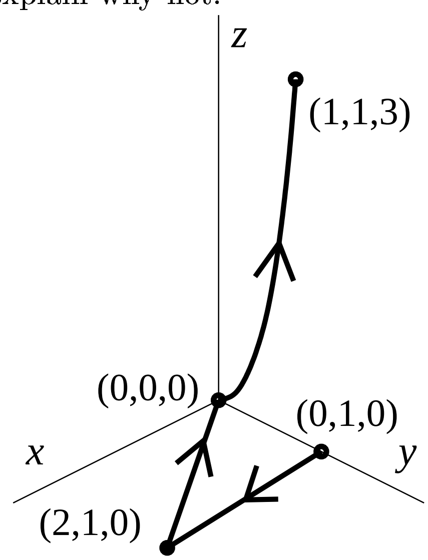


(20) 3. Suppose $\mathbf{F}(x, y, z) = \left(4y^5e^{4x} + 3z\right)\mathbf{i} + \left(5y^4e^{4x}\right)\mathbf{j} + (3x + 2z)\,\mathbf{k}$, a vector field defined and continuously differentiable throughout $\mathbb{R}^3$.

a) Determine whether there is a scalar function $P(x, y, z)$ defined everywhere in $\mathbb{R}^3$ such that $\nabla P = \mathbf{F}$. If there is such a P, find it; if there is not, explain why not.

b) Compute the integral $\int_C \mathbf{F} \cdot d\mathbf{s}$, where C is the path shown to the right. It is a line segment from $(0, 1, 0)$ to $(2, 1, 0)$ followed by a line segment from $(2, 1, 0)$ to $(0, 0, 0)$ followed by the parametric curve $\langle t, t, 3t^2 \rangle$ for $0 \le t \le 1$ which connects $(0, 0, 0)$ to $(1, 1, 3)$.

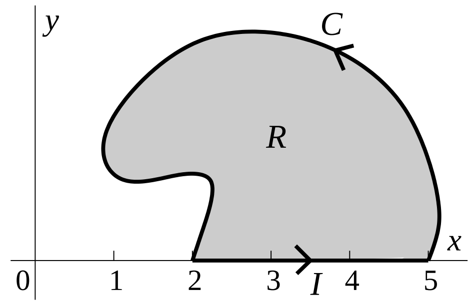
Use information gotten from your answer to a) to help if you wish. Some methods are easier than others.



(20) 4. The three-dimensional parametric curve C is defined by the equations $\begin{cases} x = \cos(t) \\ y = \sin(t) \\ z = t^2 \end{cases}$.

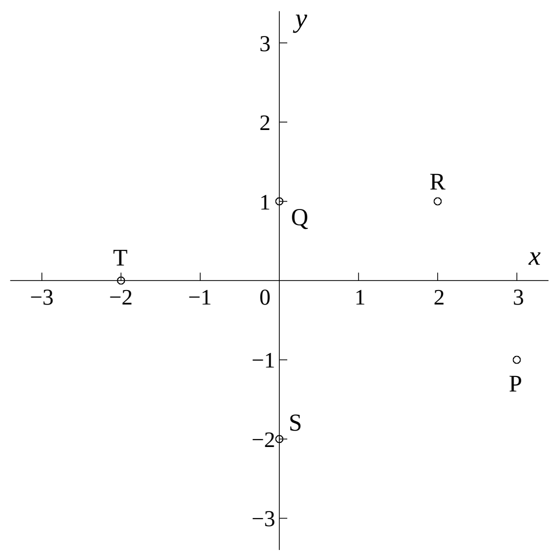
a) Verify that the curvature is given by the formula $\kappa(t)=\sqrt{\frac{5+4t^2}{(4t^2+1)^3}}$.
b) Explain briefly why $\lim\limits_{t\to\infty}\kappa(t)=0$.
c) The first two coordinates of this curve describe uniform circular motion. Explain why this statement is consistent with the limit evaluated in b). (You may wish to use pictures to help your explanation.)

(20) 5. A region R in $\mathbb{R}^2$ is located in the first quadrant, as shown. Its boundary, oriented counterclockwise as shown, is an interval $I=[2,5]$ on the x-axis and a curve C in the first quadrant.
Suppose the following information is also known:
$\iint_R 1\,dA=5$; $\iint_R x\,dA=12$; $\iint_R y\,dA=8$.



Find $\int_C \left(x^2+xy+3y\right)dx+\left(\arctan(y^3)+3x^2+2xy+x\right)dy$.
Hint $I+C$ is a positively (counterclockwise) oriented piecewise smooth simple closed curve which is the boundary of R. Be careful because the formulas for $P(x,y)=x^2+xy+3y$ and $Q(x,y)=\arctan(y^3)+3x^2+2xy+x$ together have seven "pieces".

(20) 6. Sketch the three level curves of the function $W(x,y)=x-y^2$ which pass through the points $P=(3,-1)$ and $Q=(0,1)$ and $R=(2,1)$. Be sure to label each curve with the appropriate function value and be sure that your drawing is clear and unambiguous.
Also sketch on the same axis the vectors of the gradient vector field ∇W at the points P and Q and R and S and T. The point $S=(0,-2)$ and $T=(-2,0)$.



(20) 7. Suppose $\mathbf{F}(x,y,z)=\left(x+3xz^2+e^{zy^5}+x^3\right)\mathbf{i}+\left(y^3-17z^9\right)\mathbf{j}+\left(\arctan(xy^4)+z^2\right)\mathbf{k}$, a vector field defined and continuously differentiable throughout $\mathbb{R}^3$.
Compute the total flux outward of $\mathbf{F}$ through the sphere of radius 2 centered at $(0,0,0)$. Some methods are easier than others. Be careful with your initial computations because the formula for $\mathbf{F}$ has eight "pieces"!

(20) 8. a) Suppose z is defined implicitly as a function of x and y by the equation $z^2y-3xy^2+xz=3$. Find $\frac{\partial z}{\partial x}$ at the point $p=(1,2,3)$.
b) Suppose $f(x,y,z)=z^2y-3xy^2+xz$. If $p=(1,2,3)$, find the maximum rate of change of f at p and find a vector in the direction of this maximum increase.

c) Find an equation for the plane tangent to $f(x,y,z) = z^2y - 3xy^2 + xz$ at the point $p = (1,2,3)$.

(20) 9. A bounded region in the *first octant* of $\mathbb{R}^3$ has the surface defined by $x + y + z^2 = 1$ as part of its boundary. The remainder of its boundary is given by portions of the planes $x = 0$, $y = 0$, and $z = 0$. Compute the triple integral of z over this region in space.
You probably should begin by sketching the region.

(12) 10. Define $f(x,t) = \frac{e^{\left(\frac{kx^2}{t}\right)}}{\sqrt{t}}$ where k is a constant. There is one value of k for which this function is a solution of the Heat Equation, $f_{xx} = f_t$. Find the value of k and verify that the resulting function does solve the equation.

(8) 11. Find and classify all critical points of $f(x,y) = x^2 + x + 2xy + y$.

Some formulas for the final exam in Math 251:1, 2, & 3, spring 2010

Curvature κ is all of the following:

$$\left\|\frac{d\mathbf{T}}{ds}\right\| = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \frac{\|\mathbf{r}'(t)\times\mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3} \overset{2\ \mathrm{dim}}{=} \frac{|y''(t)x'(t) - x''(t)y'(t)|}{(x'(t)^2+y'(t)^2)^{3/2}} \overset{y=f(x)}{=} \frac{|f''(x)|}{(1+(f'(x))^2)^{3/2}}$$

Second derivative test for differentiable functions in $\mathbb{R}^2$

Suppose $f_x(a,b)=0$ and $f_y(a,b)=0$. Let $H=H(a,b)=f_{xx}(a,b)f_{yy}(a,b)-[f_{xy}(a,b)]^2$.

a) If $H>0$ and $f_{xx}(a,b)>0$, then $f(a,b)$ is a local minimum.

b) If $H>0$ and $f_{xx}(a,b)<0$, then $f(a,b)$ is a local maximum.

c) If $H<0$, then $f(a,b)$ is not a local maximum or minimum (f has a saddle point).

If $H=0$, no information.

Polar coordinates $dA = r\,dr\,d\theta$

Spherical coordinates $dV = \rho^2\sin\phi\,d\rho\,d\theta\,d\phi$

Change of variables in 2 dimensions

$$\iint_{R_{xy}} f(x,y)\,dA = \iint_{\tilde{R}_{uv}} f(x(u,v),y(u,v))\left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv$$ where the Jacobian, $\frac{\partial(x,y)}{\partial(u,v)}$, is $\det\begin{pmatrix}\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v}\\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}\end{pmatrix}$.

Green's Theorem

$$\int_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

If $\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$ and $\mathbf{F}$ is a vector field then $\begin{cases}\text{curl } F = \nabla\times\mathbf{F}, \text{ a vector field.}\\ \text{div } F = \nabla\cdot\mathbf{F}, \text{ a function.}\end{cases}$

Stokes' Theorem

S is a surface with boundary curve C. As you "walk" along C, S is to the left and $\mathbf{N}$, the surface normal, is up.

$$\left[\iint_S (\text{curl } \mathbf{F})\cdot\mathbf{N}\,dS = \right] \quad \iint_S (\nabla\times\mathbf{F})\cdot d\mathbf{S} = \int_C \mathbf{F}\cdot d\mathbf{s} \quad \left[= \int_C P\,dx + Q\,dy + R\,dz\right]$$

Divergence Theorem

W is a region in $\mathbb{R}^3$ with boundary surface S. The boundary S is oriented so its normal vectors point *outward*.

$$\iint_S \mathbf{F}\cdot d\mathbf{S} = \iiint_E \text{div } F\,dV \quad \left[= \iiint_E \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}\,dV\right]$$

A **A**

Final Exam for Math 251, sections 1, 2, & 3

May 6, 2010

NAME ____

Do all problems, in any order.

Show your work. An answer alone may not receive full credit.

No notes and no calculators may be used on this exam.

A sheet with some formulas is the last page of the exam.

"Simplification" of answers is not necessary unless otherwise stated, but standard values of traditional functions such as e^0 and $\sin\left(\frac{\pi}{2}\right)$ should be given.

Problem Number	Possible Points	Points Earned:
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	
9	20	
10	12	
11	8	
Total Points Earned:		

A **A**