Math 251:1-2-3 **Workshop Problems #3** **March 30, 2010**

1. Rewrite the integral

$$\int_0^1 \int_0^{2x} \int_0^{3y} H(x, y, z)\, dz\, dy\, dx$$

as a $dx\, dy\, dz$ integral.

Note The integral can't be computed since no information about H is given. You probably should begin by sketching, as accurately as possible, the volume over which the triple iterated integral is taken.

2. Some of these iterated integrals correspond to real geometric problems (computation of volumes) and some do not. Some are actually illegal! Please indicate which are good and which are not. Explain your answers. Compute any iterated integral which corresponds to a volume. Include a sketch of each volume computed.

a) $\displaystyle\int_0^{x^3} \int_0^{y} x^4 + y^2 + 7\, dx\, dy$ b) $\displaystyle\int_0^1 \int_0^{5x} x^4 + y^2 + 7\, dx\, dy$

c) $\displaystyle\int_5^7 \int_{y^3}^{3y} x^4 + y^2 + 7\, dx\, dy$ d) $\displaystyle\int_{-1}^0 \int_{2y}^{-y^2} x^4 + y^2 + 7\, dx\, dy$

3. Compute:

a) $\displaystyle\int_0^{\frac{\pi}{2}} \left(\int_y^{\frac{\pi}{2}} \frac{\sin x}{x}\, dx \right) dy$ b) $\displaystyle\int_0^1 \left(\int_{\sqrt{y}}^1 e^{(7x^3)}\, dx \right) dy$ c) $\displaystyle\int_0^1 \left(\int_x^{x^{1/3}} \sqrt{1 - y^4}\, dy \right) dx$

Hint Write 'em as a double integral, then re-iterate*.

4. Find the volume lying between the two paraboloids $z = x^2 + y^2$ and $\mathbf{3}z = 4 - x^2 - y^2$. (Make a rough sketch of the region involved and then write the volume as a double integral over an area in the $xy-$plane.)

5. Compute $\displaystyle\int_1^2 \int_x^{x^2} \int_z^{zx} xyz\, d?_1\, d?_2\, d?_3$ where $?_1$ and $?_2$ and $?_3$ stand for some ordering of the variables x and y and z which you must determine. (Only one of the 6 possible selections "makes sense", though!)

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* Yes, this is supposed to be a somewhat incomprehensible clue.