Formulas for the final exam in Math 251:12-14 and 15-17, fall 2010

Curvature $\kappa$ is all of the following:
\[
\left\| \frac{d\mathbf{T}}{ds} \right\| = \left\| \frac{\mathbf{T}'(t)}{\left| \mathbf{r}'(t) \right|} \right\| = \left| \frac{\mathbf{r}'(t) \times \mathbf{r}''(t)}{\left| \mathbf{r}'(t) \right|^3} \right| 2 \dim \frac{\left| y''(t)x'(t) - x''(t)y'(t) \right|}{\left( x'(t)^2 + y'(t)^2 \right)^{3/2}} \] $y = f(x)$
\[
= \frac{\left| f''(x) \right|}{\left( 1 + (f'(x))^2 \right)^{3/2}}
\]

Second derivative test for differentiable functions in $\mathbb{R}^2$
Suppose $f_x(a, b) = 0$ and $f_y(a, b) = 0$. Let $H = H(a, b) = f_{xx}(a, b)f_{yy}(a, b) - [f_{xy}(a, b)]^2$.

a) If $H > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local minimum.

b) If $H > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local maximum.

c) If $H < 0$, then $f(a, b)$ is not a local maximum or minimum ($f$ has a saddle point).
If $H = 0$, no information.

Polar coordinates $dA = r \, dr \, d\theta$
Spherical coordinates $dV = \rho^2 \sin \phi \, d\rho \, d\theta \, d\phi$

Change of variables in 2 dimensions
\[
\int \int_{R_{xy}} f(x, y) \, dA = \int \int_{R_{uv}} f(x(u, v), y(u, v)) \left| \frac{\partial(x, y)}{\partial(u, v)} \right| \, du \, dv
\]
where the Jacobian, $\frac{\partial(x, y)}{\partial(u, v)}$, is det \[
\begin{pmatrix}
\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\
\frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}
\end{pmatrix}
\]

Green’s Theorem
\[
\int_C P \, dx + Q \, dy = \int \int_R \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \, dA
\]

If $\nabla = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k}$ and $\mathbf{F}$ is a vector field then \[
\begin{cases}
\text{curl } \mathbf{F} = \nabla \times \mathbf{F}, \text{ a vector field.} \\
\text{div } \mathbf{F} = \nabla \cdot \mathbf{F}, \text{ a function.}
\end{cases}
\]

Stokes’ Theorem
$S$ is a surface with boundary curve $C$. As you “walk” along $C$, $S$ is to the left and $\mathbf{N}$, the surface normal, is up.
\[
\left[ \int \int_S \left( \text{curl } \mathbf{F} \right) \cdot \mathbf{N} \, dS \right] = \int \int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = \int_C \mathbf{F} \cdot d\mathbf{s} = \int_C P \, dx + Q \, dy + R \, dz
\]

Divergence Theorem
$W$ is a region in $\mathbb{R}^3$ with boundary surface $S$. The boundary $S$ is oriented so its normal vectors point outward.
\[
\int \int_S \mathbf{F} \cdot d\mathbf{S} = \int \int \int_W \text{div } \mathbf{F} \, dV = \int \int \int_E \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \, dV
\]