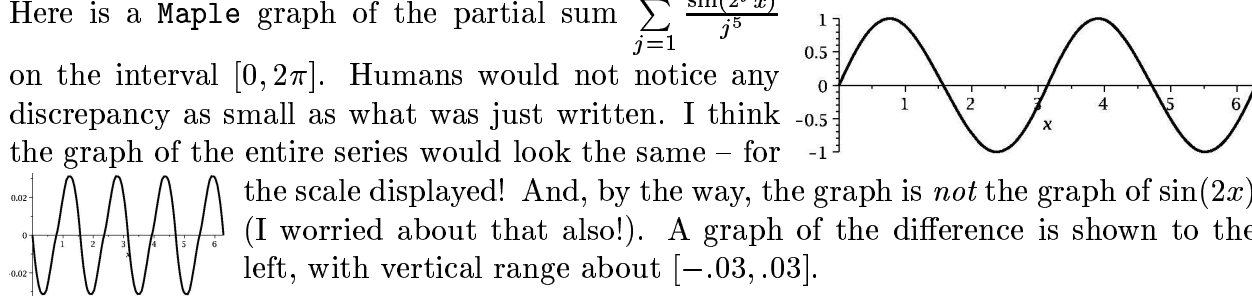


Here are graphs of the partial sums of some trigonometric series.

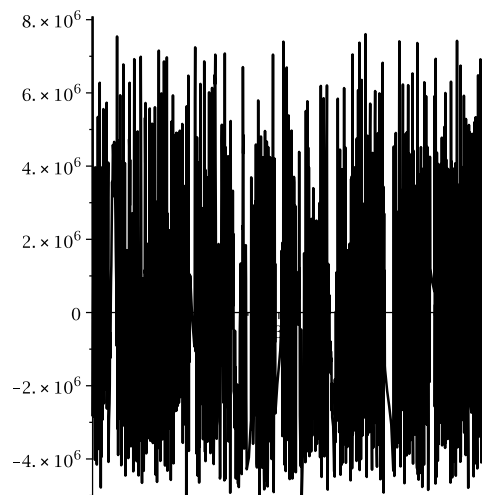
If w is real, then $|\sin w| \leq 1$ so the infinite series $\sum_{j=1}^{\infty} \frac{\sin(2^j x)}{j^5}$ converges for all x because it converges absolutely. An easy estimate with an improper integral ($\sum_{j=51}^{\infty} \frac{1}{j^5} < \int_{50}^{\infty} \frac{dx}{x^5}$) shows that the partial sum of the first 50 terms is within .00000004 (yes!) of the whole sum.

Here is a Maple graph of the partial sum $\sum_{j=1}^{50} \frac{\sin(2^j x)}{j^5}$ on the interval $[0, 2\pi]$. Humans would not notice any discrepancy as small as what was just written. I think the graph of the entire series would look the same – for the scale displayed! And, by the way, the graph is *not* the graph of $\sin(2x)$ (I worried about that also!). A graph of the difference is shown to the left, with vertical range about $[-.03, .03]$.



Now *here* is a Maple graph of $\sum_{j=1}^{50} 2^j \frac{\cos(2^j x)}{j^5}$, a partial sum of a much stranger trig series. I just differentiated each term in the partial sum of the first series and added the results. Notice that the graph is emphatically *not nice*!

The interval of x 's is again $[0, 2\pi]$ but Maple has scaled the vertical axis. It is about $[-5 \cdot 10^6, 8 \cdot 10^6]$. The graph isn't reliable here because of the difficulties of plotting a function with such wiggly behavior.



The series $\sum_{j=1}^{\infty} 2^j \frac{\cos(2^j x)}{j^5}$ probably doesn't converge very often. For example, suppose $x = \frac{\pi}{2^k}$ for some positive integer, k . Then as soon as the index of summation, j , passes k , the cosine function is evaluated at a multiple of 2π so the terms of the series 'are $\frac{2^j}{j^5}$, which does *not* $\rightarrow 0$. Many other x 's have similar or even more bizarre properties. Therefore the term-by-term derivative of the original series does not converge.

I am **not** asserting that the sum of the original series is not differentiable (it is related to yet another "special function" which Maple identifies and can differentiate). I do assert that the generalization of "The derivative of a sum is the sum of the derivatives" to an infinite series is false in this case. Power series are very nice. Trigonometric series are not.