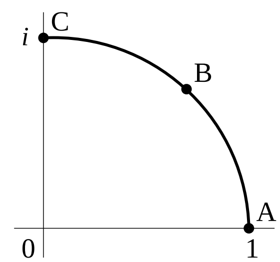
Math 403:01 **Homework assignment #2** **February 1, 2010**

Due at the beginning of class, Monday, February 8, 2010

Solve these textbook problems (4 points each): 1.4 (page 41): 19, 22, 31; 1.5 (page 53): 11, 13, 14.

Also hand in solutions to the following two problems:

A. (10 points) This problem concerns z^z. The picture shows the upper right quarter of the unit circle (where $|z| = 1$, $\mathrm{Re}(z) \geq 0$, and $\mathrm{Im}(z) \geq 0$ <u>or</u> where $z = e^{it}$ and t is ...).



a) Find all values of z^z at A (where $z = 1$), B (where $z = \frac{1+i}{\sqrt{2}}$), and C (where $z = i$).

b) Suppose you start with the standard value of 1^1 for z^z at A, and move *continuously* on the quarter circle shown, computing z^z (this should probably be called the *principal branch* of z^z). What value of z^z is obtained at B? What value of z^z is obtained at C?

c) Sketch the image of this quarter circle in $\mathbb{C}$ under the mapping $z \mapsto z^z$ (the principal branch!).

Comment Remember the *definition* of A^B in this course (made this week!). Use a graphing device to sketch the curve requested in c). You can get a formula for z^z when $z = e^{it}$ in terms of familiar functions, but the formula will combine them in strange ways, and the resulting curve is strange to me. Maybe that's because z^z is, indeed, very strange.

B. (10 points) A frog starts at the origin. It leaps one unit eastward (to the right!) on its first jump, $\frac{1}{2}$ unit on its second, $\frac{1}{4}$ unit on its third, $\frac{1}{8}$ on the fourth, and so on, each time turning exactly an angle θ to the left from the previous jump. Assuming only that $0 < \theta < \pi$, show that this frog will always end up at some point on a semicircle of radius $\frac{2}{3}$. Sketch this semicircle.

Hint Example 27 on page 39 will be useful if you write the frog's travel using complex numbers.