

Math 027

Review Exercises for the Final Exam

The following are review exercises for the Math 027 final exam. These exercises are provided for you to practice or test yourself for readiness for the final exam. There are many more problems appearing here than would be on the final. These exercises represent many of the types of problems you would be expected to solve on the final, but are not meant to represent all possible types of problems that could appear on the final exam. Note for the final exam you must show **all** your work in order to receive full credit.

Scientific calculators are permitted: graphing calculators or calculators with QWERTY keyboards are NOT permitted. Word problems must be done using algebraic methods to receive full credit.

1. Evaluate the following:

a) $6 - [4(3 - 5)^2 - 7]$

b) $7^0 - [3(-4) - 5(-6)^2]$

c) $\frac{4 - 6 - 3^2}{(2 - 5)^2 + 2}$

d) $5^0 + 5^{-1} + 5^2$

2. Evaluate

a) $(xy)^2 - (x - y)^2$ if $x = 2$, and $y = -3$

b) $\frac{|y - x| - y^2}{y - x}$ if $x = 2$, and $y = -3$

3. Given $\sigma = \sqrt{\frac{1 - \rho^2}{n - 1}}$, compute σ to three places for $\rho = 0.4$ and $n = 100$.

Round your answer to the nearest thousandth.

4. Evaluate Z when $Z_1 = 0.6$, $Z_2 = 0.45$, $N_1 = 50$ and $N_2 = 70$. Round your answer to the nearest tenth.

$$Z = \frac{\frac{Z_1 - Z_2}{\frac{1}{N_1} - \frac{1}{N_2}}}{\frac{1}{N_1} - \frac{1}{N_2}}$$

5. A wholesaler pays \$22.50 for an item and marks up the price 15%. A retailer buys the item from the wholesaler and marks it up an additional 18%. What is the final cost to the consumer who buys the item from the retailer? *Round off to the nearest cent after each mark up.*

6. Gene packs B boxes per hour for 9 hours and then packs 6 boxes per hour for H hours.

a) Express the total number T of boxes that Gene packs in terms of B and H .

b) Use this expression to compute T when $B = 20$ boxes per hour and $H = 15$ hours.

7. Solve for x :

a) $2x - (7 - 4x) = 10x - (x - 2)$

b) $5x^2 - 4(x - 3) = 5x(x + 1) - 9x$

c) $\frac{2x+3}{5} - \frac{x}{3} = \frac{2x+7}{15}$

d) $\frac{x}{4} + \frac{3x-4}{10} = x - 4$

8. Solve the inequality and sketch the solution on the number line.

a) $5(x - 1) - 8x \geq 3 - x$

b) $-1 \leq 2x - 3 < 11$

9. A technology company has 17 college graduates for every 7 high school graduates in its work-force. If there are 336 in their work-force, how many college graduates are there in that company?

10. If there are 2.54 cm in an inch, to the nearest tenth, how many inches are in 20 cm?

11. Solve for p : $3(m + 2p) = 4(2p - m) + 9$

12. Determine whether or not the point $(-1, 2)$ lies on the graph of the equation

$$\frac{y}{4} - \frac{x}{2} = x + 2$$

Explain your answer.

13. Sketch the graph of the following equations: give the coordinates of the x - and y -intercepts of the graphs.

a) $3x - 5y = 15$

b) $5x + 4y = 20$

c) $4x + 7y = 28$

14. Find the slope of the line passing through the points $(-2,5)$ and $(2,-4)$.

15. Find the slope of the line given by the equation $3x - 5y = 10$.

16. Find the equation of the line passing through the points $(-2,5)$ and $(2,-4)$.

17. Find the equation of the line passing through the point $(2,-4)$ and parallel to the graph of the equation $4x + 5y = 10$.

18. The table shows (lifetime) peptic ulcer rates (per 100 population) for various education levels.

Education level (years) x	Ulcer Rate y
8	12
11	12.5
12	11
15	10
16	8
18	9

a) Plot the data on the graph.

b) Draw a line of best fit on the graph; that is, draw a **straight** line which comes as close as possible to all the data points.

c) Find the equation of this line.

19. An airline determines that the number of seats sold in the Newark to LA flight is linearly related to the price of the ticket. When all 150 seats are sold, the price is \$275 per ticket; but when 100 seats are sold the price is \$400 per ticket.

a) Write an equation relating the number of seats sold (S) to the price of a ticket (P).

b) Use this equation to find the number of seats sold if the price of a ticket is \$350.

20. Solve the system of equations algebraically.

$$\text{a) } \begin{cases} 2x + 3y = 16 \\ x - 2y = 1 \end{cases}$$

$$\text{b) } \begin{cases} 5m + n = 5 \\ m = 2n + 12 \end{cases}$$

21. Simplify completely:

$$\text{a) } (x - 3)^2 - 4(2x + 3)$$

$$\text{b) } (x - 2)(2x + 3) - (x + 3)^2$$

$$\text{c) } (x - 5)(x + 5) - (x - 5)^2$$

22. Carla takes a trip from town A to town B. She drives to town B at a rate of 55 miles per hour. She then takes a plane, which travels at a rate of 175 miles per hour, back to town A. If the total traveling time is 10 hours, to the nearest mile, what is the distance between the two towns?

23. A secretary and a trainee are processing a pile of 124 forms. The secretary processes 15 forms per hour while the trainee processes 7 forms per hour. If the trainee begins working at 9:00A.M. and is then joined by the secretary 2 hours later, at what time will they finish the pile of forms?

24. The average cost, A , to print x calendars is given by the rational function

$$A = \frac{3.25x + 850}{x}.$$

a) Using at least 5 values of x between 0 and 100, plot at least 5 points on the given axes to sketch the graph of this function. NOTE: *The graph is NOT a straight line.*

b) What is the average cost for 50 calendars?

25. The cost of 7 cassettes and 11 diskettes is \$19.32, while the cost of 2 cassettes and 4 diskettes is \$5.94. Find the cost of each cassette and each diskette?

26. Julie invests \$22,000 in two instruments: a CD which gives 5.5% interest per year, and a bond which gives 6% interest per year. If the annual interest from the two investments totals \$1257.50 per year, how much is invested at each rate?

27. Reduce to the lowest terms: $\frac{x^2 - 25}{2x^2 - 3x - 35}$

28. Perform the indicated operations and simplify:

a) $\frac{5}{3x} + \frac{3}{4x} - \frac{x+3}{12x}$

b) $\frac{x^2 + 2x}{x^2 - x - 2} \cdot \frac{x-2}{x}$

c) $\frac{3}{x-1} - \frac{2x}{x+4}$

d) $\frac{2x^2 + x - 15}{x^2 - 9} \div \frac{2x^2 - 3x - 5}{6x^2 + 7x + 1}$

29. Simplify and write your answer with *positive exponents only*:

a) $(x^{-2}y^3)(-2x^{-3}y^2)^{-2}$

b) $\frac{2x^{-2}y^{-4}}{8x^3y^{-6}}$

c) $\left(\frac{2x^{-4}y^3}{-6x^2y^{-2}}\right)^{-2}$

30. Simplify completely. All radicals must be in simplest radical form.

a) $\sqrt{24} - \sqrt{96}$

b) $\sqrt{5}(\sqrt{2} + \sqrt{5})$

c) $\frac{15}{\sqrt{3}}$

d) $\sqrt{3} + \frac{6}{\sqrt{3}}$

31. Decide if the following is true or false and why:

$$\sqrt{a^2 + b^2} = a + b$$

(Justify your answer to receive full credit.)

32. Solve the following for x :

a) $x^2 + 10 = 7x$

b) $3x^2 - 14x - 5 = 0$

c) $2x^2 - 5x + 1 = 0$

33. The population of a town decreases by 27 people each day. If the town's population is 110,235 today,

a) What will the population be 123 days from today?

b) What will the population be t days from today?

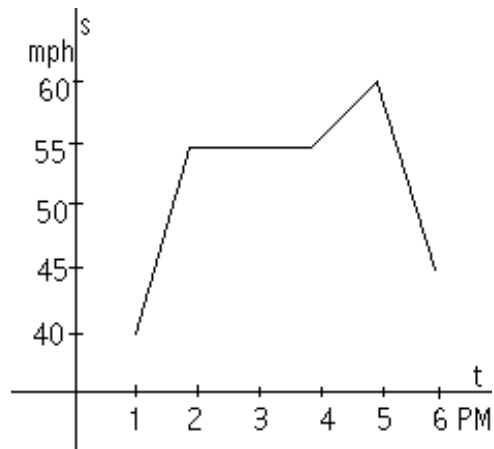
34. A population of bacteria in a particular medium doubles each hour. If you start with a culture of 500 bacteria (at 0 hour),

a) Determine a formula that represents the bacteria population P after t hours.

b) Use the formula you found above to determine how many bacteria are present at 24 hours.

35. Explain the difference between arithmetic and geometric sequence and give an example of each.

- 36.** The number of bacteria in a culture that initially contained 3000 doubles every 40 minutes. What is the number of bacteria present after 200 minutes?
- 37.** Determine if the following sequences are arithmetic, geometric, or neither.
- $\{5, 15, 45, 135, \dots\}$
 - $\{-3, -7, -11, -15\}$
 - $\{x_n\}$ where x_n is defined by $x_n = 3n - 4$
- 38.** A car is now worth \$23,000.00, but loses 13% of its value each year.
- Find a formula which expresses the value (V) of the car at year t .
 - Use the formula you found in part (a) to find the value of the car in 12 years.
- 39.** \$5,000.00 is placed in a bank earning interest at 7% per year.
- Find a formula which expresses the value (A) of the account at year t .
 - Use the formula you found in part (a) to find the value of the account in 10 years.
- 40.** The graph illustrates Jane's speed during a 5-hour drive. The horizontal axis, t , represents the time of day, and the vertical axis, s , represents her speed at that time.



- How much does her speed increase from 1 to 2 PM?
- During what time period is her speed constant?
- What is this constant speed?
- During what time period does her speed decrease?

41. A telephone company has two plans:

Plan 1 - 5c per minute all 24 hours a day and 7 days a week anywhere in USA, with a monthly fee of \$5.95.

Plan 2 - 12c per minute all 24 hours a day and 7 days a week anywhere in USA, with no monthly fee.


a) Under what conditions is one plan better than the other? Show analysis – You must use mathematics (graph and/or equations) to support your answer.

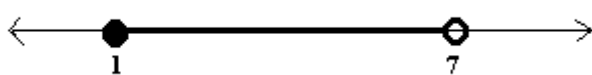
b) For how many minutes a month do both the plans cost the same?

ANSWERS: MATH 027 FINAL EXAM REVIEW EXERCISES

1. a) -3 b) 193 c) -1 d) $26\frac{1}{5}$ 2. a) 11 b) $\frac{4}{5}$ 3. 0.092 4. 26.3 5. 30.53

6. a) $T = 9B + 6H$ b) $T = 270$ 7. a) $x = -3$ b) No Solution c) $x = 2$ d) $x = 8$

8. a) $x \leq -4$ 

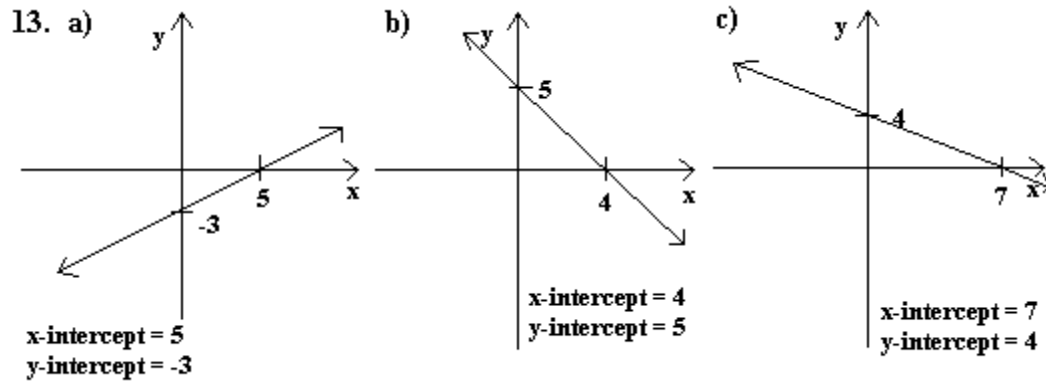
b) $1 \leq x < 7$ 

9. 238 college graduates 10. 7.9 inches 11. $p = \frac{7m-9}{2}$ 12. Let $x = -1$ and $y = 2$. Then

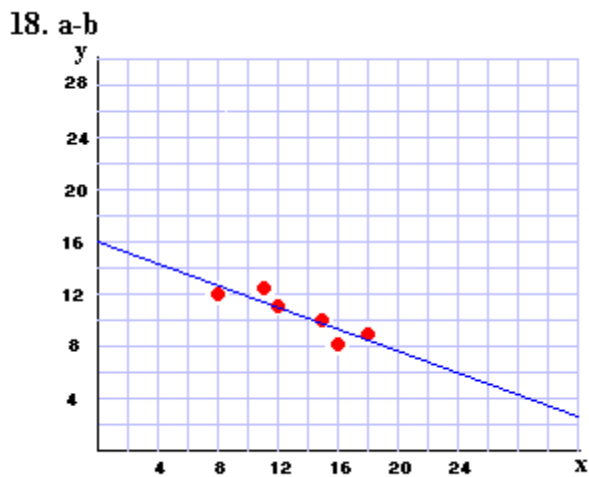
$\frac{y}{4} - \frac{x}{2} = \frac{2}{4} - \frac{-1}{2} = 1$ and $x + 2 = -1 + 2 = 1$. Since $\frac{y}{4} - \frac{x}{2} = x + 2$ for $x = -1$ and $y = 2$, then

the ordered pair $(-1, 2)$ satisfies the equation $\frac{y}{4} - \frac{x}{2} = x + 2$. Therefore the *point* $(-1, 2)$ lies of

the graph of the equation $\frac{y}{4} - \frac{x}{2} = x + 2$.



14. $-\frac{9}{4}$ 15. $\frac{3}{5}$ 16. $y = -\frac{9}{4}x + \frac{1}{2}$ 17. $y = -\frac{4}{5}x - \frac{12}{5}$

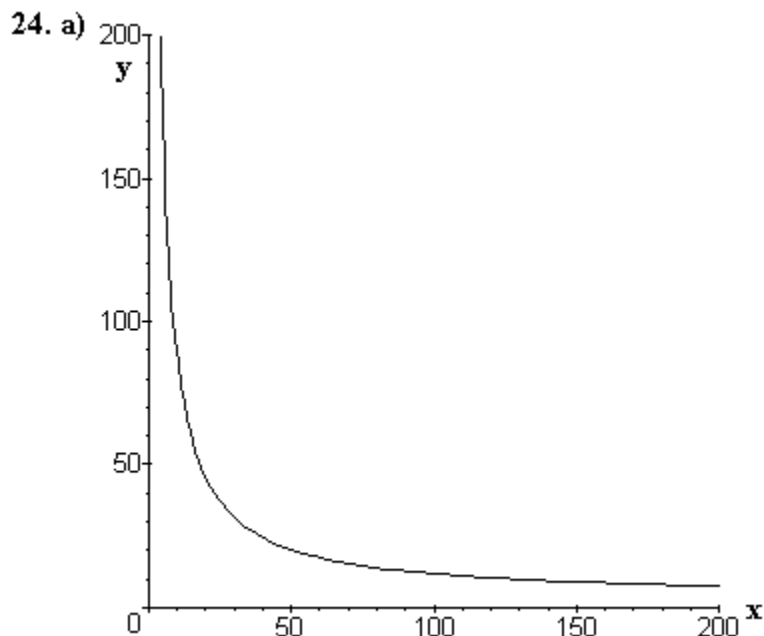


Note that this line is an estimate and will probably vary as will the equation.

18. c) Using the points (0,16) and (24,6) on the line drawn above, the equation of the line is $y = -\frac{5}{12}x + 16$.

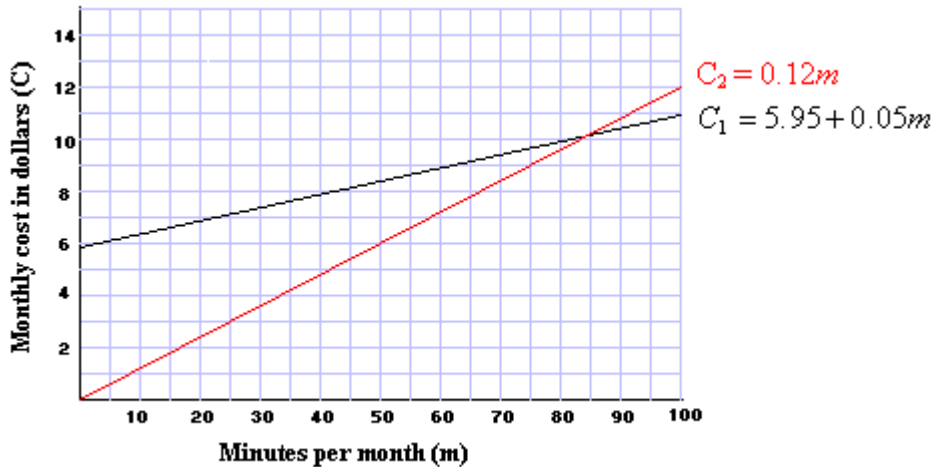
19. a) $P = -\frac{5}{2}s + 650$ b) 120 seats 20. a) $x = 5, y = 2$ b) $m = 2, n = -5$

21. a) $x^2 - 14x - 3$ b) $x^2 - 7x - 15$ c) $10x - 50$ 22. 418 miles 23. 4:00 PM



24. b) \$20.25 25. Cost of 1 cassette = \$1.99, cost of 1 diskette = \$0.49 26. \$12,500 in the CD and \$9,500 in the bond 27. $\frac{x+5}{2x+7}$ 28. a) $\frac{26-x}{12x}$ b) $\frac{x+2}{x+1}$ c) $\frac{-2x^2+5x+12}{(x-1)(x+4)}$
- d) $\frac{6x+1}{x-3}$ 29. a) $\frac{x^4}{4y}$ b) $\frac{y^2}{4x^5}$ c) $\frac{9x^{12}}{y^{10}}$ 30. a) $-2\sqrt{6}$ b) $5+\sqrt{10}$ c) $5\sqrt{3}$
- d) $3\sqrt{3}$ 31. False. Let $a=2$ and $b=3$. Then $\sqrt{a^2+b^2} = \sqrt{2^2+3^2} = \sqrt{4+9} = \sqrt{13}$ but $a+b=2+3=5$. Since $\sqrt{2^2+3^2} \neq 2+3$, then $\sqrt{a^2+b^2} \neq a+b$
32. a) $x=2, 5$ b) $x=-1/3, 5$ c) $x = \frac{5 \pm \sqrt{17}}{4}$ 33. a) 106,914 b) $110,235 - 27t$
34. b) $P = 500 \cdot 2^t$ b) 8,388,608,000 bacteria 35. An arithmetic sequence is a sequence in which the difference between any two successive terms is constant. A geometric sequence is a sequence in which the ratio between any two successive terms is constant. An example of an arithmetic sequence is $\{3, 10, 17, 24, 31, \dots\}$; an example of a geometric sequence is $\{3, 6, 12, 24, 48, \dots\}$.
36. 96,000 bacteria 37. a) geometric b) arithmetic c) arithmetic
38. a) $V = 23,000(0.87)^t$ b) \$4,324.73 39. a) $A = 5,000(1.07)^t$ b) \$9,835.76
40. a) 15 mph b) Between 2 PM and 4 PM c) 55 mph d) Between 5 PM and 6 PM

41. The monthly cost for plan 1, C_1 , is found by $C_1 = 5.95 + 0.05m$ where m is the number of minutes used for the month: the monthly cost for plan 2, C_2 , is found by $C_2 = 0.12m$ where m is the number of minutes used for the month. The graph for both plans are shown below:



As you can see by the graph, Plan 2 is better (less expensive) if you plan to use less than 85 minutes per month: Plan 1 is less expensive if you plan to use more than 85 minutes per month.

If you solve the system $\begin{cases} C_1 = 5.95 + 0.05m \\ C_2 = 0.12m \end{cases}$, you will find the point at which both plans are the same: (85, 10.20) which means that at 85 minutes both plans cost the same: \$10.20.