

Math 111

Review Exercises for the Final Exam

The following are review exercises for the Math 111 final exam. These exercises are provided for you to practice or test yourself for readiness for the final exam. There are many more problems appearing here than would be on the final. These exercises represent many of the types of problems you would be expected to solve on the final, but are not meant to represent all possible types of problems that could appear on the final exam.

Your final exam will be in two parts: the first part does not allow the use of a calculator, and the second part does allow the use of a graphing calculator. Since the exercises in this review sheet are mixed together, we have put a \square symbol next to exercises or parts of exercises where you WILL be allowed to use the graphing calculator: otherwise you should be able to solve the problem WITHOUT a calculator. Such a symbol will not be on the final exam. Please note that for the final, you may use any graphing calculator except the TI-89, TI-Inspire, and any calculator with a QWERTY keypad.

Show all your work: unsupported results may not receive credit.

1. Perform the operations and express your answer in simplest form with positive exponents only:

a) $x^{-5} \left(\frac{4x^{-2}y^3}{3x^{-5}} \right)^{-2}$

b) $2y^3 \left(\frac{12x^3y^{-2}}{3x^{-5}y^4} \right)^{1/3}$

c) $\frac{3(1+x)^{1/2} - x(1+x)^{-1/2}}{1+x}$

2. Perform the operations and express your answer in simplest form.

a) $\frac{x-5}{x^2-2x-8} - \frac{x+1}{x+2}$

b) $\frac{x-4}{3x^3-27x} - \frac{2x+1}{x-3}$

3. Express the following as a simple fraction reduced to lowest terms: $1 - \frac{4}{z} + \frac{4}{z^2}$
 $\frac{1}{z^2} - \frac{2}{z^3}$.

4. Solve for y in the equation: $\frac{1}{y} + \frac{1}{x} = \frac{1}{z}$

5. Simplify the radical: (LEAVE IN RADICAL FORM)

a) $\sqrt[3]{54x^5y^9z^4}$

b) $\sqrt{24x^3y^2z^5} \sqrt{12xyz^2}$

c) $\sqrt[3]{x^2} \sqrt{x}$

6. Rationalize the denominator and simplify:

a) $\frac{8x}{\sqrt[3]{2xy^2}}$

b) $\frac{6}{\sqrt{10} - \sqrt{7}}$

c) $\frac{x-3}{\sqrt{x} - \sqrt{3}}$

d) $\frac{3h}{\sqrt{x-h} - \sqrt{x}}$

7. Solve for x: $|x+3| - 2 = 8$

8. Solve the inequality: and graph the solution on the number line:

a) $|3x-2| < 7$

b) $|3x-4| \geq 3$

9. An entertainment system was purchased for \$3,000 in 1997. A linear relationship was used to determine that the value of the system in 1999 was \$1,500.
- Express the value, v , as a function of time, t , in years.
 - Use your function to predict how much would you expect the value to be in the year 2000.
 - Use your function to determine in which year the entertainment system will be worth nothing.

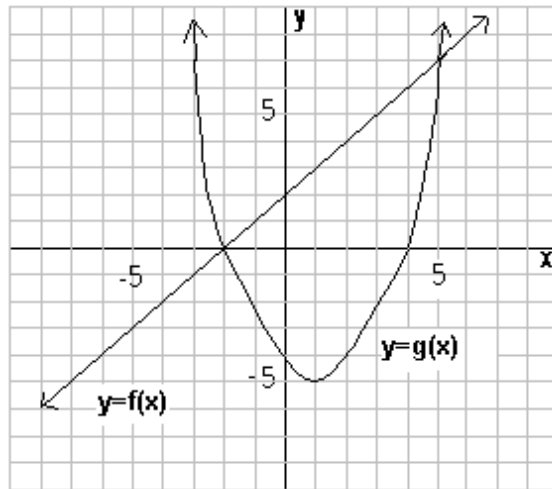
10. Given the points A (3,1), B (-1,5), C (4,5) and D (2,7). M is the midpoint of AB and N is the midpoint of CD.
- Find the distance between M and N.
 - What is the equation of the circle having the center M and radius 5?
 - What is the equation of a circle having a center of A, and containing the point B?
 - What is the equation of a circle with diameter having endpoints C and D?

11. Use the following data to answer the questions below.

x	y
0	4.0
1	2.7
2	2.1
3	1.1
4	0.0

- Plot the data points on a graph.
 - Draw a line of best fit (the regression line) on the graph.
 - Find the equation of this line.
12. Jason has invested in two investments: a CD that earns 4% per year and a bond that earns 8% per year. He invested \$3,520 more in the CD than the bond. How much did he invest in the CD and the bond if he receives a total of \$1,100.80 in annual interest from his two investments?
13. Show that $2 - i$ is a solution to $x^2 - 4x = -5$.
14. Express $\frac{3 - 2i}{1 + i}$ as a number in the form $a + bi$.
15. Solve for x in the quadratic equation $9x^2 + 6x + 5 = 0$ and give your answer in simplest complex form.

16. The graph of $y = f(x)$ and $y = g(x)$ are shown.



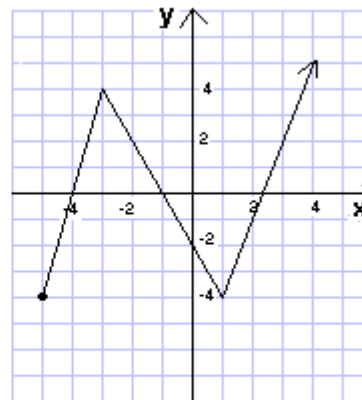
- Evaluate $(f + g)(2)$.
- Evaluate $f(g(1))$.
- Find the interval(s) of x where $g(x) > f(x)$.

17. Given $f(x)$ defined by the table below left, and $g(x)$ defined by the graph below right:

$f(x)$ defined by

x	$f(x)$
-1	3
0	1
1	-2
2	-3

$g(x)$ defined by the graph below:



Find

- $f(2)$
- $(f \circ g)(2)$
- $g(f(1))$

18. Given $f(x) = 2x^2 - x + 1$,

a) Evaluate $f(-2)$.

b) Find and simplify $\frac{f(x+h) - f(x)}{h}$.

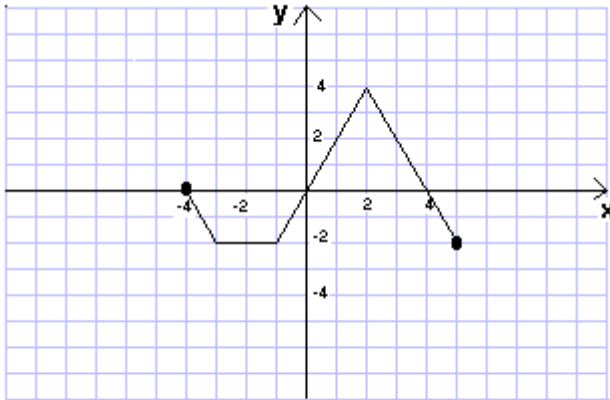
19. Given $f(x) = \frac{5x}{x+1}$, find and simplify $\frac{f(x+h) - f(x)}{h}$.

20. Find the DOMAIN of the following functions: express your answer using interval notation. You must show your analysis to receive full credit.

a) $f(x) = \sqrt{4 - x^2}$

b) $f(x) = \sqrt{\frac{x+3}{x-4}}$

21. The graph of $y = f(x)$ appears below.



a) Sketch the graph of $f(x-2) + 1$ on the same set of axes as above.

b) Sketch the graph of $-f(x) + 2$ on the same set of axes as above.

c) If $h(x) = f(x-2) + 1$, then evaluate $h(3)$.

22. Given $f(x) = (x-1)^2 + 2$

a) Sketch the graph of $y = f(x)$.

b) Find the domain of $f(x)$.

c) Find the range of $f(x)$.

d) Find the y-intercept(s).

e) Find the coordinates of the vertex.

23. Given $f(x) = \sqrt{x-1} - 2$
- Sketch the graph of $y = f(x)$.
 - Find the domain of $f(x)$.
 - Find the range of $f(x)$.
 - Find the x -intercept(s).

24. Given $f(x) = \begin{cases} 2x+1 & \text{if } x \leq -1 \\ 3-x^2 & \text{if } x > -1 \end{cases}$

- Evaluate $f(2)$.
- Evaluate $f(-2)$.
- Sketch the graph of $f(x)$.

25. Given $f(x) = \begin{cases} x+1 & \text{if } x < 0 \\ x^2-1 & \text{if } 0 \leq x < 2 \\ 4 & \text{if } x \geq 2 \end{cases}$

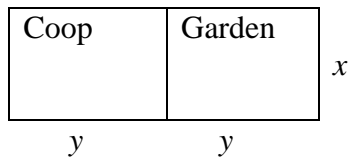
- Evaluate the following: $f(-2), f(0), f(1), f(2), f(10)$
- Sketch the graph of $f(x)$.

26. In economics, the demand function for a given item indicates how the price per unit p (in dollars) is related to number of units x that are sold. Suppose that a company finds that the demand function for one of the items it produces is

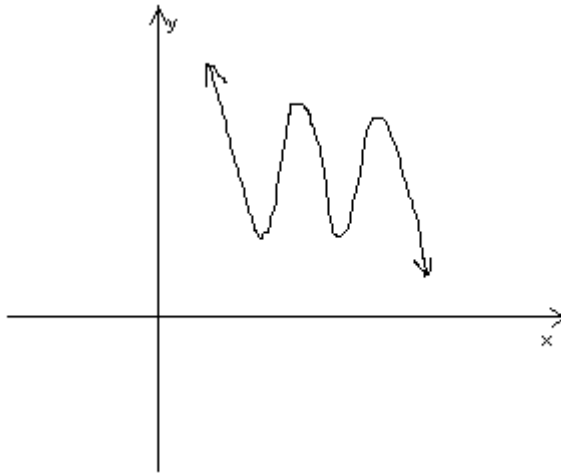
$$p = 20 - 0.05x$$

- The revenue function, $R(x)$, is found by multiplying the price per unit p and number of units sold x : that is $R(x) = px$. Write the formula for the revenue function in terms of x .
- How many units should be produced in order to get the maximum revenue? (Show your work.)
- What is the corresponding unit price for your answer in part (b) above? (Show your work.)

27. A farmer wants to fence in an area for his chicken coop and for a small garden as shown in the diagram below with exactly 200 linear feet of fencing as shown in the diagram, with width x .



- Express the area of the coop A , as a function of x .
 - Find the value of x which will give the rectangular garden with greatest area.
 - What is the maximum area?
28. Given the function $f(x) = (x + 1)(x - 2)(x + 3)$
- Graph the polynomial
 - What is the degree of the function?
 - Describe the end behavior.
29. The graph of the polynomial function $y = f(x)$ is given below.



- What is the smallest possible degree of $f(x)$?
- Is it possible for $f(x)$ to have degree 8? Explain your answer.
- From the above graph what can you tell about the leading coefficient of $f(x)$?

30. Sketch of the graph of the following functions. Identify the x -intercept(s). Identify the y -intercept(s). Identify the horizontal asymptote(s). Identify the vertical asymptote(s).

a) $f(x) = \frac{x-1}{(x-2)(x+3)}$

b) $f(x) = \frac{2x^2}{x^2-9}$

31. Solve the equations for x : Give your answer to the nearest hundredth.

a) $x^4 = 2x + \sqrt{x} + 5$

b) $x + \frac{1}{x} = 7x^4$

32. For the function $y = x^3 - 7x^2 + 3x + 25$.

- a) Find the local maximum values for on the interval $[0,6]$ to the nearest tenth.
- b) Find the local minimum value(s) on the interval $[0,6]$ to the nearest tenth.
- c) Could this function have two local maximums over the entire interval? Explain your answer.

33. Solve the inequality $x^3 \geq 7 - 4x^2$ accurate to one decimal place. Express your answer using interval notation.

34. Given the function $f(x) = .01x^5 - 2x + 1$:

- a.) Draw the graph on the viewing rectangle: $[-6,6]$ by $[-6,6]$.
- b.) Identify all zeros of $g(x)$ accurate to one decimal place.
- c) Use the graph to solve the inequality $.01x^5 - 2x + 1 > 0$ accurate to one decimal place. Express your answer using interval notation.

35. Given $f(x) = \frac{x+4}{3-2x}$

- a) Find $f^{-1}(x)$.
- b) Find $f(3)$.
- c) Find $f^{-1}(3)$
- d) Find $f \circ f^{-1}(3)$

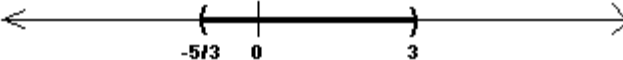
36. Given $f(x) = x^3 + x + 4$, find $f^{-1}(1)$ accurate to 2 decimal places.

ANSWERS: MATH 111 FINAL EXAM REVIEW EXERCISES

1. a) $\frac{9}{16x^{11}y^6}$ b) $2^{5/3}x^{8/3}y$ c) $\frac{3+2x}{(1+x)^{3/2}}$ 2. a) $\frac{-x^2+4x-1}{(x-4)(x+2)}$ b) $\frac{-6x^3-21x^2-8x-4}{3x(x-3)(x+3)}$

3. $z(z-2)$ 4. $\frac{xz}{x-z}$ 5. a) $3xy^3z\sqrt[3]{2x^2z}$ b) $12x^2yz^3\sqrt{2yz}$ c) $x\sqrt[6]{x}$

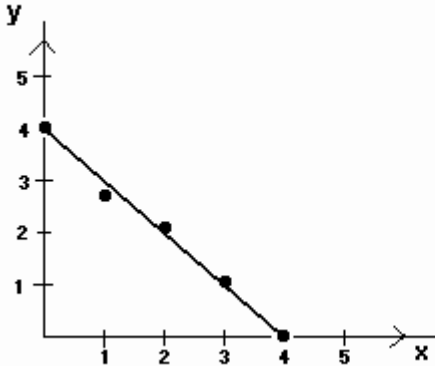
6. a) $\frac{4\sqrt[3]{4x^2y}}{y}$ b) $2\sqrt{10}+2\sqrt{7}$ c) $\sqrt{x}+\sqrt{3}$ d) $-3\sqrt{x-h}-3\sqrt{x}$ 7. $x = -13, 7$

8. a) $-5/3 < x < 3$  A number line with arrows at both ends. There are tick marks at -5/3, 0, and 3. Open parentheses are placed above the tick marks at -5/3 and 3, and a line segment connects them.

b) $x \leq 1/3$ or $x \geq 7/3$  A number line with arrows at both ends. There are tick marks at 1/3 and 7/3. Closed brackets are placed above the tick marks at 1/3 and 7/3, and lines extend from them to the left and right respectively.

9. a) $v = -750t + 1,500,750$ b) \$750 c) 2001 10. a) $\sqrt{13}$ b) $(x-1)^2 + (y-3)^2 = 25$
c) $(x-3)^2 + (y-1)^2 = 32$ d) $(x-3)^2 + (y-6)^2 = 2$

11 a-b



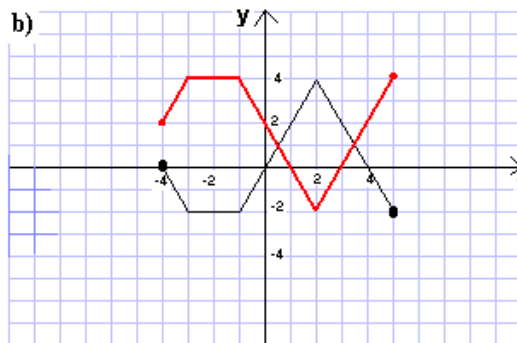
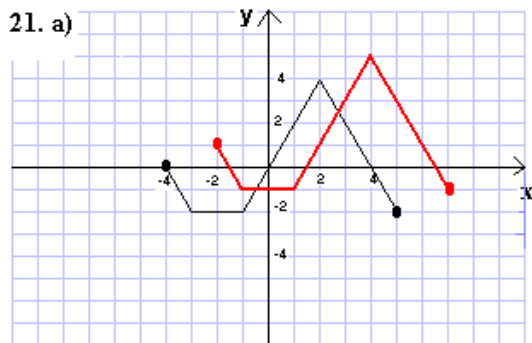
c) $y = -x + 4$

12. \$11,520 invested in the CD; \$8,000 invested in the bond.

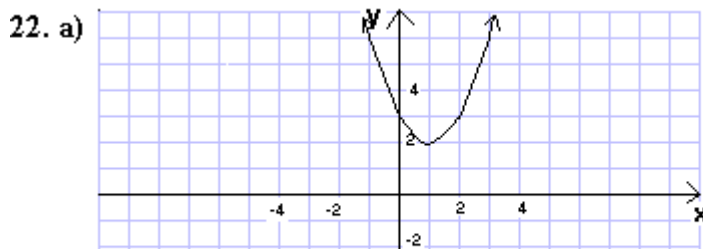
13. $(2-i)^2 - 4(2-i) = 4 - 4i + i^2 - 8 + 4i = 4 - 4i - 1 - 8 + 4i = -5$ 14. $\frac{1}{2} - \frac{5}{2}i$

15. $-\frac{1}{3} \pm \frac{2}{3}i$ 16. a) 0 b) -3 c) $(-\infty, -2) \cup (5, \infty)$ 17. a) -3 b) 3 c) 2

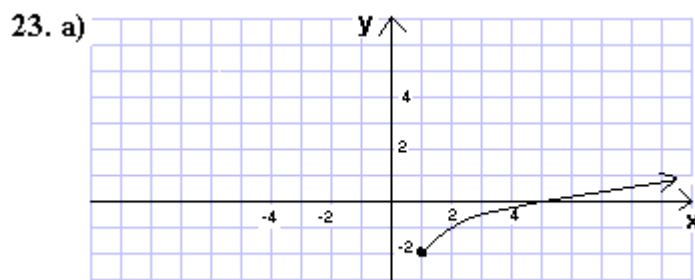
18. a) 11 b) $4x + 2h - 1$ 19. $\frac{5}{(x+h+1)(x+1)}$ 20. a) $[-2, 2]$ b) $(-\infty, -3] \cup (4, \infty)$



c) 3

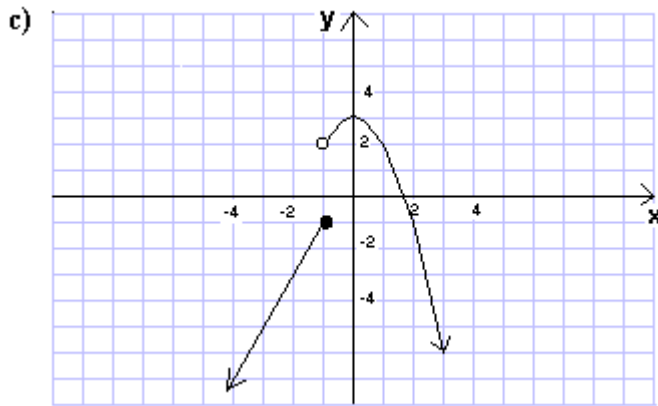


b) $(-\infty, \infty)$ c) $[2, \infty)$ d) $(0, 3)$ e) $(1, 2)$

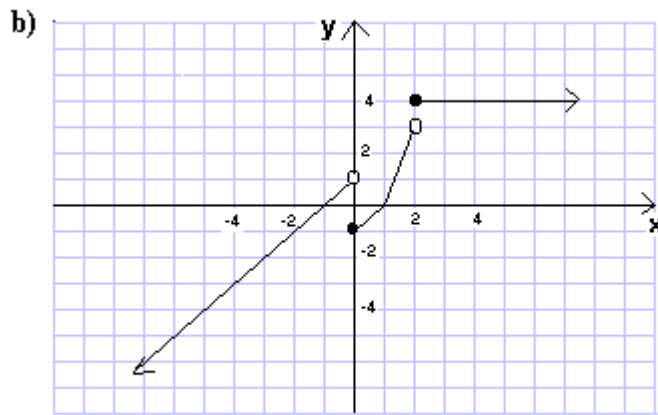


b) $[1, \infty)$ c) $[-2, \infty)$ d) $(5, 0)$

24. a) -1 b) -3

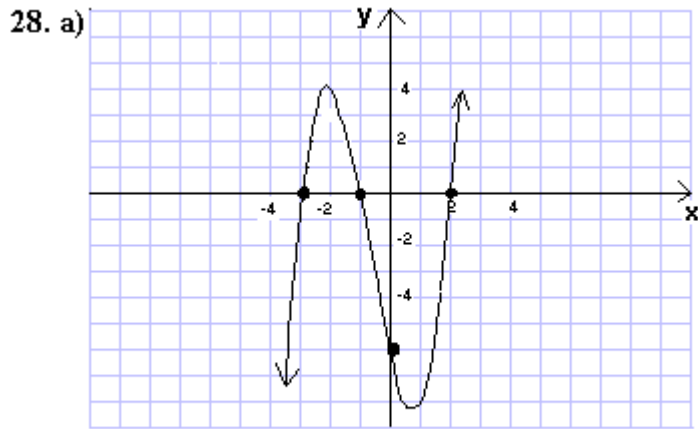


25. a) -1, -1, 0, 4, 4



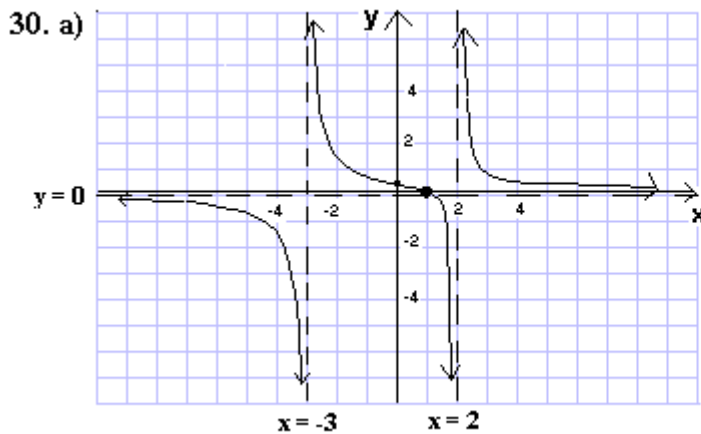
26. a) $R(x) = (20 - 0.5x)x = -0.05x^2 + 20x$ b) 200 units c) \$10

27. a) $A = \frac{1}{4}x(200 - 3x) = -\frac{3}{4}x^2 + 50x$ b) $33\frac{1}{3}$ ft c) $833\frac{1}{3}$ sq ft



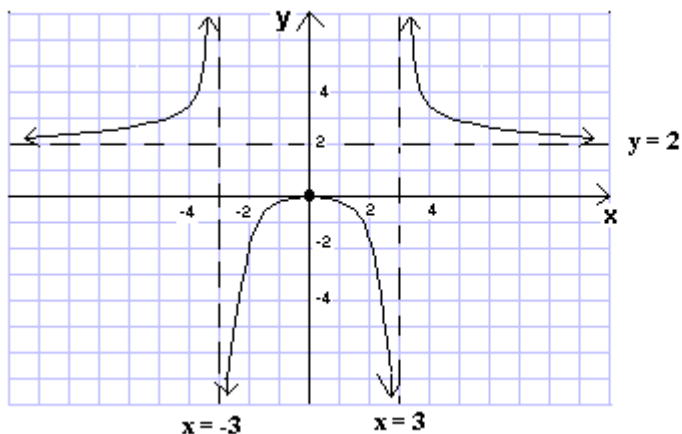
b) 3 c) As $x \rightarrow \infty, y \rightarrow \infty$: As $x \rightarrow -\infty, y \rightarrow -\infty$

29. a) 5 b) No, the end behavior indicates that the function has odd degree. c) The leading coefficient must be negative.



x-intercept = 1: y-intercept = 1/6
Horiz. asymptote: $y=0$
Vert. Asymptotes: $x = -3, x = 2$

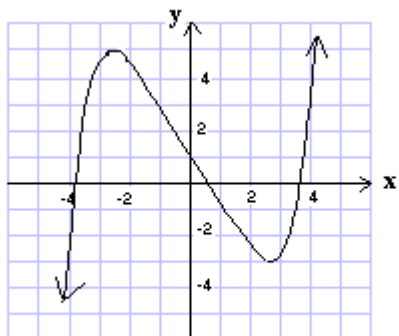
30. b)



x-intercept=0: y-intercept=0
 Horiz. Asymptote: $y=2$
 Vert. Asymptotes: $x=-3, x=3$

31. a) $x = 1.77$ b) $x = 0.74$ 32. a) 25.3 b) -12.1 c) No. Since it is a cubic function, it cannot have more than 2 turning points: one local max and one local min. 33. $[-3.4, -1.8] \cup [1.2, \infty)$

34. a)



b) zeros: -3.9, 0.5, 3.6 c) $(-3.9, 0.5) \cup (3.6, \infty)$ 35. a) $f^{-1}(x) = \frac{3x-4}{2x+1}$ b) $-7/3$
 c) $5/7$ d) 3 36. $f^{-1}(1) = -1.21$