Solutions for review problems

7.

$$\frac{1}{\frac{\pi}{4}}\int_0^{\frac{\pi}{4}} \tan^2 x dx = \frac{4}{\pi} - 1$$

6.

a) convergent to 1

b) divergent

c) convergent to $2\sqrt{2}$

d) divergent. (Hint : use substitution $u = \arctan x$)

5.

$$\int_0^{50} 3x dx + \int_0^{50} 200 dx = \frac{3}{2}50^2 + 10000$$

4

$$\int_0^1 \sqrt{1 + (-\frac{1}{2}e^{-\frac{x}{2}})^2} = \int_0^1 \sqrt{1 + \frac{1}{4}e^{-x}} dx$$

3. Use intergration by parts

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 2\pi x(0 - \cos x) dx = 4\pi^2$$

2.

b) rotating along y axis: by cylinderical shell method,

$$\int_0^2 2\pi x(\sqrt{8 + 2x^2} - 2x) dx = \frac{\pi}{3}2^5$$

a)rotating along x axis: by cross section method,

$$\pi \int_0^2 \sqrt{8 + 2x^2}^2 - (2x)^2 dx = \pi\frac{2^5}{3}$$

1.

f)

$$\frac{1}{4}\cos 2x - \frac{1}{24}\cos 12x + C$$ Hint: use the formula $\sin A \cos B$ on the formula sheet

e)

$$\frac{1}{4}\ln|\csc(\sin^{-1}\frac{x}{4}) - \cot(\sin^{-1}\frac{x}{4})| + C$$

d)

$$\ln|x + 1| + \frac{1}{2}\arctan\frac{x}{2} + C$$

c)

$$\frac{1}{2}(\frac{1}{5}\sin^5 2x - \frac{2}{7}\sin^7 2x + \frac{1}{9}\sin^9 2x) + C$$

b)

$$\frac{1}{3}x^3 \arctan x - \frac{1}{6}x^2 + \frac{1}{6}\ln|1 + x^2| + C$$

a)

$$\frac{x^2}{3}e^{3x} - \frac{2x}{9}e^{3x} + \frac{2}{27}e^{3x} + \frac{2}{3}e3x + C$$