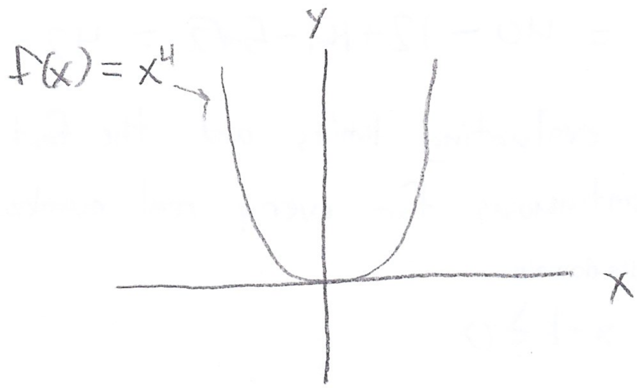
Math 151–Spring 2011
Quiz 1 Name: ______________________ Section: __________

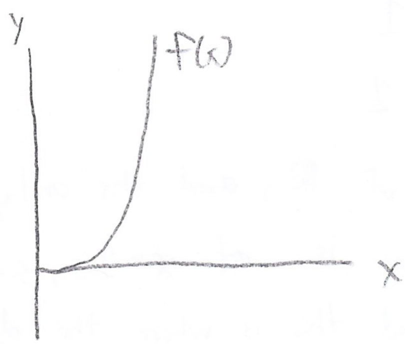
Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible; sketch all relevant graphs and write down all relevant mathematics. You have 18 minutes to take this 15 point quiz.

1. (*5 points*) Find a domain over which $f(x) = x^4$ is one-to-one, and find $f^{-1}(x)$ over that domain. Sketch the graphs of both functions.



Note that this is not 1:1 over the whole real line, as it does not pass the horizontal line test.

If we restrict the domain to $[0, \infty)$, we get



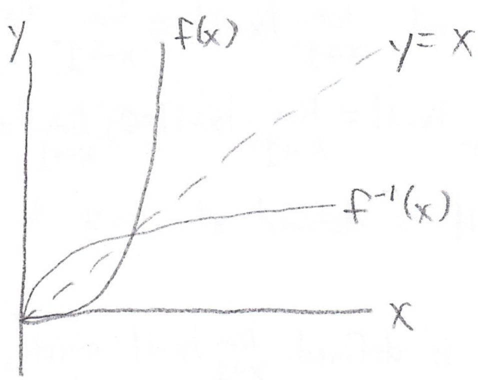
This is 1:1.

To find $f^{-1}(x)$: $y = x^4$

Take 4th roots: $y^{\frac{1}{4}} = x$

So $f^{-1}(x) = x^{\frac{1}{4}}$.

One finds the graph of $f^{-1}(x)$ by reflecting $f(x)$ over the line $y=x$:



2. (*5 points*) Evaluate $\lim_{x\to 2}(5x^3 - 3x^2 + 7x - 5\sqrt{x})$.

$$\lim_{x\to 2}(5x^3-3x^2+7x-5\sqrt{x}) = \lim_{x\to 2} 5x^3 - \lim_{x\to 2} 3x^2 + \lim_{x\to 2} 7x - \lim_{x\to 2} 5\sqrt{x}$$

$$= 5\lim_{x\to 2} x^3 - 3\lim_{x\to 2} x^2 + 7\lim_{x\to 2} x - 5\lim_{x\to 2}\sqrt{x}$$

$$= 5(2^3) - 3(2^2) + 7(2) - 5(\sqrt{2})$$

$$= 40 - 12 + 14 - 5\sqrt{2} = 42 - 5\sqrt{2}$$

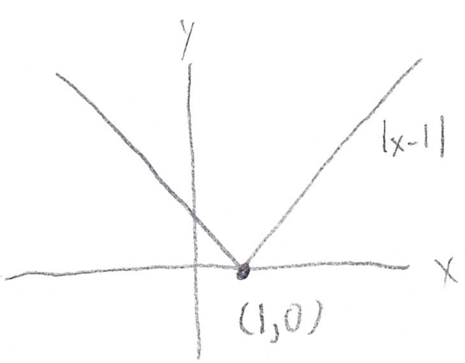
This used the rules for evaluating limits and the fact that the function x^n is continuous for every real number n.

3. (*5 points*) Prove that $|x-1|$ is continuous on its domain.

$$|x-1| = \begin{cases} x-1 & \text{when } x-1 \geq 0 \\ -(x-1) & \text{when } x-1 < 0 \end{cases}$$

$$= \begin{cases} x-1 & \text{when } x \geq 1 \\ 1-x & \text{when } x < 1 \end{cases}$$

Clearly the domain is all of $\mathbb{R}$, and the only place $|x-1|$ might not be continuous is at $x=1$, since it is linear everywhere else and this is where the definition changes.



We see that $\lim_{x\to 1^-} |x-1| = \lim_{x\to 1^-}(1-x) = 1-1 = 0$

and $\lim_{x\to 1^+} |x-1| = \lim_{x\to 1^+}(x-1) = 1-1 = 0$.

Since $\lim_{x\to 1^-} |x-1| = \lim_{x\to 1^+} |x-1| = 0$, $\lim_{x\to 1} |x-1| = 0$.

Also, $|x-1|$ is defined at $x=1$ to be $|1-1| = |0| = 0$.

So at $x=1$, $|x-1|$ is defined, $\lim_{x\to 1} |x-1|$ exists, and these two values agree. Therefore $|x-1|$ is continuous at $x=1$.