Math 151–Fall 2011
Quiz 3 Name: _____ Section: _____

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible; sketch all relevant graphs and write down all relevant mathematics. You have 15 minutes to take this 15 point quiz.

1. (*8 points*) Use implcit differentiation to find the slope of the curve $x^3y^3 + 4xy - 3x^2 + 2y = 1$ at $x = 1$.

First we implicitly differentiate:

$$\underbrace{3x^2y + x^3y'}_{\frac{d}{dx}(x^3y)} + \underbrace{4y + 4xy'}_{\frac{d}{dx}(4xy)} - \underbrace{6x}_{\frac{d}{dx}(3x^2)} + \underbrace{2y'}_{\frac{d}{dx}2y} = \underbrace{0}_{\frac{d}{dx}1}$$

Then, solve for y':

$$x^3y' + 4xy' + 2y' = 6x - 4y - 3x^2y$$

$$y'(x^3 + 4x + 2) = 6x - 4y - 3x^2y$$

$$y' = \frac{6x - 4y - 3x^2y}{x^3 + 4x + 2}$$

To find the slope at $x = 1$, we also need to know the y value.

At $x = 1$, the equation defining the curve looks like

$$1^3y + 4 \cdot 1 \cdot y - 3 \cdot 1^2 + 2y = 1$$

$$7y = 4$$

$$y = \frac{4}{7}$$

Plug in $(1, \frac{4}{7})$ to y':

$$y' = \frac{6 \cdot 1 - 4 \cdot \frac{4}{7} - 3 \cdot 1^2 \cdot \frac{4}{7}}{1^3 + 4 \cdot 1 + 2}$$

$$= \frac{6 - \frac{16}{7} - \frac{12}{7}}{7} = \frac{2}{7}$$

2. (*7 points*) Use logarithmic differentiation to find the derivative of $(1+\sin 2x)^{1+x^2}$.

$$y = (1+\sin(2x))^{1+x^2}$$

Take the logarithm of both sides.

$$\ln y = \ln\left((1+\sin(2x))^{1+x^2}\right)$$

$$\ln y = (1+x^2)\ln(1+\sin(2x))$$

Now use implicit differentiation.

$$\frac{1}{y}y' = (2x)\ln(1+\sin(2x)) + (1+x^2)\cdot\frac{1}{1+\sin(2x)}\cdot\cos(2x)\cdot 2$$

$$\frac{1}{y}y' = 2x\ln(1+\sin(2x)) + \frac{2(1+x^2)\cos(2x)}{1+\sin(2x)}$$

$$y' = \left(2x\ln(1+\sin(2x)) + \frac{2(1+x^2)\cos(2x)}{1+\sin(2x)}\right)y$$

$$y' = \left(2x\ln(1+\sin(2x)) + \frac{2(1+x^2)\cos(2x)}{1+\sin(2x)}\right)(1+\sin(2x))^{1+x^2}$$