Exam 3 Info

Date: May 6th, 2016

Location: Normal classroom (ARC-205, Busch Campus)

Time: 8-11 am

Notes:

1. The exam will be cumulative, but will be slightly more focused on the material since Exam 2 (Bendixson's criterion and PDE).
2. There will be **at least** one question taken verbatim from one of the previous two exams.
3. There will be **no** MATLAB questions on the exam.
4. You are allowed to bring **two** sheets of **handwritten** notes as a reference during the exam. I will not allow any typed sheets to be used, including the lecture notes (i.e. the main text).
5. Calculators, cell phones, computers, etc. are **prohibited**. All calculations will be able to be completed by hand.
6. I have designed the exam to take approximately two hours, but you may stay the full three if you'd prefer.

Suggestions:

1. Understand all previous exam questions.
2. Understand all assigned homework questions (solutions on Sakai).
3. Solve the Extra Credit HW # 11 and the (non-graded) Review problems (HW # 12) on the website (relating to the last PDE material).
3. Read over ES and class notes. Note that the PDE chapter was covered sporadically, so you are only responsible for the specific material discussed in class(see the Course Calendar and below for more details). Of course, the rest is interesting as well!

Material:
All material covered in the course is fair-game for the exam. Regarding material from the first two exams, I suggest looking at their respective information sheets to get an extensive list, although I will highlight some important material below. As usual, be aware: this list is **not** exhaustive, and anything covered could appear on the exam.

1. **Previous exam material**
   (a) Difference equations
      (i) Steady states
      (ii) Linear stability
      (iii) Cobwebbing

(b) ODEs

(i) **Complete** analysis of **one** dimensional ODE equations (steady states, asymptotic behavior, concavity, etc.)

(ii) Linear stability (i.e. Jacobian analysis, including eigendirections)

(iii) Non-dimensionalization

(iv) Chemostat (model formulation and dynamics)

(v) Basic growth models (exponential and logistic)

(vi) Nullcline analysis of planar systems

(vii) Epidemiology

(viii) Chemical kinetics

(ix) Enzymatic reactions (including Michaelis-Menten rate law and approximation)

(x) Simple models of transcription and translation, including effects of activation and repression.

(x) Periodic solutions (limit cycles and the Poincaré-Bendixson theorem for proving periodic orbits exist in a region $D \subset \mathbb{R}^2$).

2. **New material**

(a) Bendixson's criterion (for proving a periodic solution **cannot** exist in a region $D \subset \mathbb{R}^2$).

(b) Fundamental equation of biological motion (i.e. conservation of mass):

$$\frac{\partial c(\mathbf{x},t)}{\partial t} = -\nabla \cdot J(\mathbf{x},t) + \sigma(\mathbf{x},t). \tag{1}$$

You should understand what each term represents, in particular $J(\mathbf{x},t)$ representing movement (i.e. flux) and $\sigma(\mathbf{x},t)$ accounting for birth/death (exponential growth in particular, and the relation between the growth rate λ to the doubling time τ: $\lambda = \frac{\ln 2}{\tau}$).

(c) Transport (advection) equation: $J_t(\mathbf{x},t) = c\mathbf{v}$ in equation (1). Understand the physical interpretation as well as the solution (in one dimension, constant velocty v and exponential growth $\sigma = \lambda c$).

(d) Chemotaxis: $J_c(\mathbf{x},t) = \alpha c \nabla V$ in equation (1). Here $V = V(x)$ represents the distribution of the chemical in space. Again, understand the physical interpretation and how to use the PDE (1) to study properties of the solution (for example, show that $c(x,t)$ is increasing at a maximum of V).

(e) Diffusion: $J_d(\mathbf{x},t) = -D\nabla c$ in equation (1).

(i) What diffusion represents physically (both at the mirco- and macro-scopic level).

(ii) Separation of variables to solve one-dimensional diffusion equation (possibly with specified growth rate σ; think exponential growth $\sigma = \lambda c$).

(iii) Fixed and **no-flux** boundary conditions (for the case of **closed** tubes, which means $J(0,t) = 0$, for example at $x = 0$, as opposed to $c(0,t) = 0$. That is, no flow is allowed through at the endpoint, so we think of the boundary as having a "cap," as opposed to being open). So, in the case of one-dimensional **diffusion only**, this becomes the condition $\frac{\partial c}{\partial x}(0,t) = 0$ (since D is a positive constant). if other fluxes are present, such as chemotaxis and/or transport, this condition changes based on how the flux $J(\mathbf{x},t)$ changes. The general boundary conditions are always

$$J(0,t) = 0 \qquad \text{and/or} \qquad J(L,t) = 0,$$

depending on whether the left and/or right end is "closed" (i.e. "capped").

(f) Equations with a combination of different fluxes (for example, diffusion and chemotaxis). Be able to write down conservation of mass (equation (1)) given different biological scenarios.

(g) Systems of PDEs. I didn't really discuss this, but I expect that you are able to write equations (no analysis) for simple biological motion and the interaction of multiple populations, given biological assumptions. For example, say I have bacteria and a nutrient both diffusing in a one-dimensional chemostat. What **two** equations (one for bacteria $n(x, t)$, one for nutrient $c(x, t)$) might you posit to describe their interaction (both movement and consumption)? One such model might take the general form

$$\begin{aligned} \frac{\partial n}{\partial t}(x,t) &= D_n \frac{\partial^2 n}{\partial x^2}(x,t) + K(c(x,t))n(x,t) \\ \frac{\partial c}{\partial t}(x,t) &= D_c \frac{\partial^2 c}{\partial x^2}(x,t) - \alpha K(c(x,t))n(x,t) \end{aligned}$$

Here $K(c)$ is the (nutrient-dependent) reproduction rate of the bacteria. Furthermore, assuming Michaelis-Menten kinetics for the this growth rate, we might further use the following functional form to describe $K(c)$:

$$K(c) = \frac{k_{\max} c}{k_m + c}.$$

As with other exams, **model formulation and interpretation is important**. You should be comfortable with the arguments made to derive many of the equations, as well as the assumptions used in the various formulations. **There will be at least one question where I ask you to write down a model based on some assumptions.**