Math 250:08, Workshop 1

Solve the following problems. In the first problem, it suffices to show the necessary computations or offer brief explanations. Solutions to the other problems should be written clearly, in English sentences, incorporating mathematical symbols, equations, and diagrams where appropriate. Students are encouraged to work together on the problems during the workshop period, but all solutions must be written up individually. The first problem will be graded out of 4 points, and one other problem will be selected at random to be graded out of 6 points.

1. (a) Compute each of the following:

(i) $\begin{bmatrix} 2 & 3 & -1 \\ 1 & 0 & -5 \end{bmatrix} \left(\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} - \begin{bmatrix} 2 \\ 7 \\ -1 \end{bmatrix} \right)$

(ii) $\left(\begin{bmatrix} -3 & 1 \\ 1 & 2 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 2 \\ -1 & 2 & 0 \end{bmatrix}^T \right) \begin{bmatrix} -5 \\ 2 \end{bmatrix}$

(b) Determine whether each of the following exists. If so, simplify it; if not, explain why.

(i) $(A+B^T)^T((I_m)^T v)$, where A is an $m \times n$ matrix, B is an $n \times m$ matrix, and $v \in \mathbb{R}^m$

(ii) $(A^T+B)^T((I_m)^T v)$, where A is an $m \times n$ matrix, B is an $n \times m$ matrix, and $v \in \mathbb{R}^m$

2. Sketch each of the following sets, and determine whether each is a subspace of the appropriate $\mathbb{R}^n$. If so, prove it from the definition. If not, prove it, either by giving a counterexample to show that the definition fails to hold or by invoking a known and rigorous result from class, and describe the smallest subspace of $\mathbb{R}^n$ that contains the given set.

(a) $\{(x_1,x_2) \in \mathbb{R}^2 \,|\, 2x_1 - x_2 = 0 \text{ and } x_1 \geq 0\}$

(b) $\{(x_1,x_2) \in \mathbb{R}^2 \,|\, x_1 - x_2 = 0 \text{ or } x_1 + x_2 = 0\}$

(c) $\{(x_1,x_2,x_3) \in \mathbb{R}^3 \,|\, x_1 + x_2 - x_3 = 0 \text{ and } x_1 - 2x_2 = 0\}$

(d) $\{(x_1,x_2,x_3) \in \mathbb{R}^3 \,|\, x_1^2 + x_2^2 + x_3^2 = 4\}$

3. A new variant on chess is played on the integer lattice in three-dimensional space, that is, the infinitely many points in $\mathbb{R}^3$ whose coordinates are all integers. In this game, the directions “left” and “right” refer to positive and negative movements on the x_1-axis, respectively, “forward” and “backward” refer to the x_2-axis, and “up” and “down” refer to the x_3-axis. One of the pieces, the über-knight, can move in exactly four ways:

(i) 1 right, 1 backward, and 3 down; (i)’ 1 left, 1 forward, and 3 up;
(ii) 2 right, 1 forward, and 2 down; (ii)’ 2 left, 1 backward, and 2 up;

The über-knight starts the game at the origin, i.e. at the point $(0,0,0)$. Determine whether it’s possible for the über-knight to ever land on the point $(22,29,2)$. If so, describe a sequence of moves that would land it there. If not, explain why.