Math 250:08, Workshop 2

Solve the following problems. In the first problem, it suffices to show the necessary computations with only brief explanations. Solutions to the other problems should be written clearly, in English sentences, incorporating mathematical symbols, equations, and diagrams where appropriate. Students are encouraged to work together on the problems during the workshop period, but all solutions must be written up individually. The first problem will be graded out of 4 points, and one other problem will be selected at random to be graded out of 6 points.

1. (a) Find a subset of $S = \left\{ \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix} \right\}$ of smallest size that generates $\operatorname{Span} S$.

(b) Compute $\begin{bmatrix} 1 & 0 & -1 & 2 \\ 2 & 3 & 1 & -1 \\ 3 & -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 2 & 3 \\ 0 & -1 \\ 3 & 0 \end{bmatrix}$.

2. Each of the following statements is false. Prove this by giving specific counterexamples.
(a) Matrix multiplication is commutative on the space of $n \times n$ matrices.
(d) If A is an $m \times n$ matrix and $b \in \mathbb{R}^m$, then the solution set to $Ax = b$ is a subspace of $\mathbb{R}^n$.
(b) For all $v_1, v_2 \in \mathbb{R}^3$, $\operatorname{Span}\{v_1, v_2\}$ is a plane.
(c) If the intersection of $\operatorname{Span} S_1$ and $\operatorname{Span} S_2$ contains a line, then $S_1 \cap S_2$ is non-empty.

3. Recall the variant of chess described in the previous workshop. It is played on the integer lattice in three-dimensional space, and the directions "right" and "left" refer to positive and negative movements[1] on the x_1-axis, respectively, "forward" and "backward" refer to the x_2-axis, and "up" and "down" refer to the x_3-axis. The über-knight starts the game at $(0,0,0)$ and can move in exactly four ways:

(i) 1 right, 1 backward, and 3 down; (i)' 1 left, 1 forward, and 3 up
(ii) 2 right, 1 forward, and 2 down; (ii)' 2 left, 1 backward, and 2 up

The pawn starts the game at $(6,8,0)$ and can move in only one way:

(a) 1 right, 1 forward, 1 up

Determine whether it's possible for the über-knight and the pawn to ever land on the same point.

[1] The "right" and "left" were mistakenly written in the reverse order on the first workshop; for this one, use the orientation described here.