

Syllabus for Oral Qualifying Exam

James Dibble

Major Topic: Riemannian Geometry

Differential geometry

- Smooth manifolds
- Vector bundles
- Lie brackets and derivatives
- Submersions, immersions, and embeddings
- Integration and Stokes' Theorem
- de Rham cohomology

Riemannian geometry

- Riemannian metrics
- First and second variation formulas
- Riemannian connections
- Geodesics and the exponential map
 - Gauss Lemma
 - Hopf-Rinow Theorem
- Parallel transport
- Jacobi fields
- Conjugate points and the cut locus
- Riemannian curvature tensor
- Sectional, Ricci, and scalar curvatures
- Second fundamental form and the shape operator

Comparison geometry

- Sectional curvature comparison
 - Gauss-Bonnet Theorem
 - Sturm and Riccati comparison inequalities
 - Cartan-Hadamard Theorem
 - Bonnet's Theorem
 - Synge's Theorem
 - Toponogov's Theorem
- Ricci curvature comparison
 - Meyers' Theorem
 - Volume estimation
 - Maximal diameter rigidity

References:

Lee, John. *Introduction to Smooth Manifolds*.
Lee, John. *Riemannian Manifolds: An Introduction to Curvature*.
Petersen, Peter. *Riemannian Geometry*.
Meyer, Wolfgang. *Toponogov's Theorem and Applications*.

Minor Topic: Algebraic Topology

Fundamental group

- Homotopy invariance
- Seifert-Van Kampen Theorem

Covering spaces

- Lifting properties
- Deck transformations
- Universal covering space

Singular homology and cohomology

- Degree theory for spheres
- Euler characteristic
- Exact sequences and excision
- Mayer-Vietoris sequence

References:

- Lee, John. *Introduction to Topological Manifolds*.
- Hatcher, Allen. *Algebraic Topology*.