

Asymptotic notation

These are used for f, g functions of some parameter, e.g. n or x , which, as here, is often suppressed in the notation. The limiting statements are meant as the parameter approaches some limit (most often $n \rightarrow \infty$); the others are for the parameter in some range (with the limit or range for the parameter either specified or clear from the context).

$$f \sim g: f/g \rightarrow 1$$

$$f = O(g): |f|/|g| \text{ is bounded above}$$

$$f = o(g): f/g \rightarrow 0$$

$$f = \Omega(g): g = O(f)$$

(equiv: $|f|/|g|$ is bounded below by a positive constant)

$$f = \omega(g): |f|/|g| \rightarrow \infty \text{ (equiv: } g = o(f))$$

$$f = \Theta(g): f = O(g) \text{ and } g = O(f)$$

(equiv: $|f|/|g|$ lies between two positive constants)

$$f \lesssim g: \limsup f/g \leq 1 \text{ (not sure we'll see this one)}$$

We can then, for example, write simply $O(g)$ to mean any (perhaps unspecified) function whose absolute value is known to be bounded above by Cg for some fixed C . Big and little “Oh” are often used for error terms, for example

$$e^x = 1 + x + O(x^2) \quad \text{as } x \rightarrow 0,$$

in which case the functions $O(\cdot)$, $o(\cdot)$ will often be negative. But more often than not f and g will be positive.