Abstract

We present new estimates obtained jointly with J. Bourgain, and partially with P. Mironescu. Each one has a different flavour, but, in fact, they are closely related.

The first one asserts that

$$\left| \int_\Gamma f(s) \overrightarrow{t}(s) \right| \leq C|\Gamma| \, \|\nabla f\|_{L^3} \quad \forall f$$

where $\Gamma \subset \mathbb{R}^3$ is a closed rectifiable curve, $|\Gamma|$ denotes the length of Γ, C is a universal constant and $\overrightarrow{t}$ is the tangent to Γ.

The second estimate concerns the classical system, in $\mathbb{R}^3$,

$$\begin{aligned} \text{div } u &= 0 \\ \text{curl } u &= f. \end{aligned}$$

Our new estimate asserts that

$$\|u\|_{L^{3/2}} \leq C\|f\|_{L^1}.$$

A third new estimate concerns the system

$$\Delta u = f \text{ in } \mathbb{R}^3,$$

where f is a divergence-free vector-field. Our new estimate asserts that

$$\|u\|_{L^3} \leq C\|f\|_{L^1}.$$

Such inequality are unusual because it is well-known that standard elliptic estimates fail in L^1.