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Gamow vectors and Borel summation

Abstract

We analyze the detailed time dependence of the wave function $\psi(x,t)$ for one dimensional Hamiltonians $H = -\partial_x^2 + V(x)$ where V (for example modeling barriers or wells) and $\psi(x,0)$ are compactly supported.

We show that the dispersive part of ψ (its asymptotic series in powers of $t^{-1/2}$) is Borel summable. The remainder, the difference between ψ and the Borel sum, is a convergent expansion of the form

$$\sum_{k=0}^{\infty} g_k \Gamma_k(x) e^{-\gamma_k t}$$

where Γ_k are the Gamow vectors of H, and γ_k are the associated resonances; generically, all g_k are nonzero. For large k, γ_k is proportional to $k \log k + k^2\pi^2 i/4$. (Gamow vectors are poles of the analytically continued Green's function, and they are generalized eigenfunctions of the Hamiltonian, with "purely growing" conditions at infinity.)

The effect of the Gamow vectors is visible when time is not very large, and the decomposition defines rigorously resonances and Gamow vectors in a nonperturbative regime, in a physically relevant way.

After Borel summation, the expansion is very rapidly convergent allowing a very sharp qualitative and quantitative control on the wave function for moderate or large time.

The analytic structure of ψ is perhaps surprising: in general (even in simple examples such as square wells), $\psi(x,t)$ turns out to be C^∞ in t but nowhere analytic on $\mathbb{R}^+$. In fact, ψ is $t-$analytic in a sector in the lower half plane and has the whole of $\mathbb{R}^+$ a natural boundary (singularity barrier).

Collaborators

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