Practice Exam #1   Math 251:H1

(That was last year's exam #1)

Name :

Problem Number	**Points Possible**	**Points**
1	15	
2	10	
3	10	
4	15	
5	10	
6	10	
7	10	
8	10	
9	10	
10 (Bonus)	10	
	Total	

1. [15 pts] Let C be the curve given by the vector function $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$. Find the equation of the line that has the direction of the binormal vector $\mathbf{B}$ to the curve C at the the point $(-1, 0, \pi)$.

2. [10 pts] Reparametrize the curve of equation $\mathbf{r}(t) = \langle 4\sin t, 3t, 4\cos t\rangle$ with respect to arc length measured from the point where $t = \frac{1}{5}$ in the direction of increasing t.

3. [**10 pts**] Imagine you're having a nice walk in the $x - y$ plane. The temperature at a general point (x, y) is given by $T(x, y) = x^2 - 3y^2$. You stop at the point $P = (2, 3)$ and realize you're getting pretty cold. From that point, what is the immediate direction $\mathbf{u} = (a, b)$ you should take to get the warmest? Justify your answer.

4. [15 pts] What is the equation of the plane containing the point $(2, 1, 3)$ and the line of equation $\langle 1 + t, t, 4t \rangle$?

5. [10 pts] Consider $f(x, y, w, z) = x^4 yzw$, where

$$\begin{cases} x = s\cos t \\ y = g_1(t)s^2 \\ z = \ln t + g_2(s) \\ w = st^2 + g_3(t) \end{cases}$$

Note that g_1, g_2 and g_3 are differentiable functions. Compute $\frac{\partial f}{\partial t}$.

6. **[10 pts]** Consider the function

$$f(x,y) = \begin{cases} \frac{(2x^2+y^2)\cos x}{\sqrt{x^2+y^2}} + 1 & (x,y) \neq 0 \\ a & (x,y) = 0. \end{cases}$$

Find the value of $a \in \mathbb{R}$ that makes f continuous at $(0,0)$. Prove, using the definition of the limit, that the obtained function f is continuous at the point $(0,0)$.

7. [10 pts] Using the identity $\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\|\|\mathbf{b}\| \sin\theta$, find the distance between the point $(1, 2, 3)$ and the line of equation $\langle 2, 6, 4\rangle + t\langle 1, 1, 1\rangle$.

8. [**10pts**] Consider two vectors $\mathbf{a}$ and $\mathbf{b}$. Prove the Parallelogram Law:

$$\|\mathbf{a}+\mathbf{b}\|^2 + \|\mathbf{a}-\mathbf{b}\|^2 = 2\|\mathbf{a}\|^2 + 2\|\mathbf{b}\|^2.$$

9. [10 pts] Consider the function of three variables $F(x, y, z) = x^2 + y^2 + z^2$ and let C be a curve given by vector function $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$ such that the curve lies on a level surface of F. Show that the vectors $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ are perpendicular for all t.

10. [Bonus Question: 10 pts] Consider the surface S given by the graph of $z = f(x, y)$ and let Π_0 the plane tangent to the surface S at the point $P_0 = (x_0, y_0, f(x_0, y_0))$. Find the equation of a plane Π_1 parallel to Π_0 and from a distance $d = \sqrt{[f_x(x_0, y_0)]^2 + [f_y(x_0, y_0)]^2 + 1}$ from Π_0. Is this plane uniquely determined?