Math 251:H1 Workshop #2

Due Date: 3/5/09

1. Let $\mathcal{C}$ be the intersection of the two surfaces $x^3 + 2xy + yz = 7$ and $3x^2 - yz = 1$. Find the equation of the tangent line to $\mathcal{C}$ at the point $P = (1, 2, 1)$.

2. Definition: The x intercept (resp. y and z intercept) of a plane is the intersection of the plane with the x axis (resp. the y and z axis).

   Show that the product of the x, y, and z intercepts of any tangent plane to the surface $xyz = c^3$ is a constant.

3. A certain function $f(x, y)$ is known to have partial derivatives of the form

$$\frac{\partial f}{\partial x} = 2y\cos(2x) + y^3x^2 + g(y), \qquad \frac{\partial f}{\partial y} = \sin(2x) + x^3y^2 + 4x + 1. \qquad (*)$$

   Note that g is a function of y only. Use the equality of mixed partial derivatives to find the function g up to an arbitrary constant. Then find all functions f with partial derivatives of the form (*).

4. Suppose that at the point $(2, -2)$, the directional derivative of the function $z = f(x, y)$ in the direction $-3\mathbf{i} + 4\mathbf{j}$ is equal to 3 and the directional derivative of $z = f(x, y)$ in the direction $\mathbf{i} + \mathbf{j}$ is equal to $\sqrt{2}$.

   (a) Determine the gradient of f at the point $(2, -2)$.

   (b) What is the maximum rate of change of the function f at the point $(2, -2)$.

5. Consider the function

$$f(x, y) = \begin{cases} \frac{(2x^2+y^2)\cos x}{\sqrt{x^2+y^2}} + 1 & (x, y) \neq 0 \\ a & (x, y) = 0. \end{cases}$$

   Find the value of $a \in \mathbb{R}$ that makes f continuous at $(0, 0)$. Prove, using the definition of the limit, that the obtained function f is continuous at the point $(0, 0)$.

6. Consider the surface S given by the graph of $z = f(x, y)$ and let Π_0 the plane tangent to the surface S at the point $P_0 = (x_0, y_0, f(x_0, y_0))$. Find the equation of a plane Π_1 parallel to Π_0 and from a distance $d = \sqrt{[f_x(x_0, y_0)]^2 + [f_y(x_0, y_0)]^2 + 1}$ from Π_0. Is this plane uniquely determined? If not, how many such planes exist? If the answer of the previous is more than one, give me the equation of all of the planes.