Math 251:H1 Workshop #3 Due date: 3/26/09

Hi guys. You should...

- work out the problem and have a clear plan of presentation before writing.
- be neat and write legibly.
- show all steps. Explain what you are doing in complete sentences.

1. A flat circular plate has the shape of the region $x^2+y^2 \leq 1$. The plate (including the boundary $x^2+y^2=1$) is heated so that the temperature T at any point (x,y) is given by $T = x^3 - x + 2y^2$. Locate the hottest and coldest points of the plate and determine the temperature at each of these points.

2. Find the dimensions of the box of maximum volume with its sides parallel to the coordinate planes that can be inscribed in the ellipsoid

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1.$$

3. A rectangle with length L and width W is cut into four smaller rectangles by two lines parallel to the sides.
   (a) Find the minimum value of the sum of the squares of the areas of the smaller rectangles.
   (b) Show that the maximum of the sum of the squares of the areas occurs when the cutting lines correspond to sides of the rectangle (so that there is only one rectangle).

4. Find and classify all the critical points of

$$f(x,y) = 3xy - x^2y - xy^2.$$

   Once you are done, find the absolute maximum and the absolute minimum of f on the rectangle $R = \{(x,y) \mid -2 \leq x \leq 3, -1 \leq y \leq 2\}$.

5. Given n *positive* numbers $a_1, a_2, \ldots, a_n$, find the maximum value of the expression $a_1x_1 + a_2x_2 + \cdots + a_nx_n$, if the variables $x_1, x_2, \ldots, x_n$ are restricted so that the sum of their squares is one. Also, what is the minimum value of $a_1x_1 + a_2x_2 + \cdots + a_nx_n$ in this case?

6. **Boltzmann Distribution:** Consider the constants $N, E, E_1, \ldots, E_n$ and show that there is a constant μ such that the maximum of the **entropy** S given by

$$S(x_1, x_2, \ldots, x_n) = x_1 \ln x_1 + x_2 \ln x_2 + \cdots + x_n \ln x_n$$

   subject to the constraints

$$x_1 + x_2 + \cdots + x_n = N, \qquad E_1x_1 + E_2x_2 + \cdots + E_nx_n = E$$

   occurs for $x_i = A^{-1}e^{\mu E_i}$, $i = 1, \ldots, n$, where

$$A = N^{-1}\left(e^{\mu E_1} + e^{\mu E_2} + \cdots e^{\mu E_n}\right).$$

   This result is used in physics to determine the distribution of velocities of gas molecules at temperature T; x_i is the number of molecules with kinetic energy E_i; $\mu = -(kT)^{-1}$, where $k \approx 1.38065 \cdot 10^{-23} J/K$ is the Boltzmann's constant; N is the total number of particles.