Math 115: Precalculus
Answer key

This may contain some typos. Please let me know if you find one. Thanks!

1. (a) $\angle AOB = \pi/4$.
   (b) $(\sqrt{2}/2, \sqrt{2}/2)$.
   (c) $\sec(\angle AOB) = \sqrt{2}$.

2. (a)
   - Amplitude = 2.
   - Period = $2\pi/3$.
   - Frequency = $3/(2\pi)$.
   - Phase shift = $\pi/3$.

   (b) Consult your calculator.

3. $x = 4$

4. (a) θ is in quadrant IV, so $\cos\theta = \sqrt{1 - \sin\theta^2} = \sqrt{1 - 0.32^2}$.
   (b) $\sin 2\theta = 2\sin\theta\cos\theta = 2 * 0.32 * \sqrt{1 - 0.32^2}$ and $\cos\theta = 1 - 2\sin^2\theta = 1 - 2 * 0.32^2$.

5. (a) The graph of $g(x)$ intersects each vertical line at a exactly (best) / at most (not as good) one point.
   (b) The graph of $g(x)$ intersects each horizontal line at at most (not the best, but enough) / exactly (perfect, extra credit for this point) one point.
   (c) Since $g(1) = 27$ and g is one-to-one, $g^{-1}(27) = 1$.
   (d) $g^{-1}(x) = (\sqrt[3]{x} - 1)/2$.

6. (a) $1200(\cot 35^o - \cot 50^o)$ ft.
   (b) 706.8581 ft.

7. $\sqrt{72^2 + 80^2 - 2 * 72 * 80 * \cos 70^o} \approx 87.43$ mi.

8. Note: This problem was previously mistakenly posted as solving $2\cos^2 t - 3\cos t + 2 = 0$. The correct one should be solving $2\cos^2 t - 3\cos t - 2 = 0$.
   (a) $x = -1/2$ and $x = 2$.
   (b) $x = -1/2$.

(c) $\cos t = -1/2$ and $t \in [0, 2\pi)$, so $t = 2\pi/3$ and $4\pi/3$. In fact, you don't need a calculator for this problem.

9. (a) $x = \log_{\frac{4}{27}} \frac{5}{72} = \frac{-2\log 3 + \log 5 - 3\log 2}{2\log 2 - 3\log 3}$. (Any one of them is acceptable.)
   (b) $x \approx 1.3968$.

10. Another typo. The function $f(x)$ should be $\log_2(2 + x)$.
   (a) Use your calculator. Note that the domain of $f(x)$ is $(-2, \infty)$. Viewing rectangle you can pick is $[-2, 12] \times [-4, 4]$.
   (b) -0.27, 4.21, 5.06.
   (c) $x \in (-0.27, 4.21) \cup (5.06, \infty)$.

11. $f^{-1}(x) = \frac{\ln(x-3)+1}{2}$.

12. $x = \pm\sqrt{2}$.

13. (a) $\cos x + \sin x$.
   (b) $\sin(x + \pi/4) = \frac{\sqrt{2}}{2} \sin x + \frac{\sqrt{2}}{2} \cos x$.
   (c) Use (a), (b) and do as you wish!

14. (a) The half-life is 30 years, so after 60 years, one quarter of the original sample, i.e. 15 grams, remains.

   (b) The rate of decay is
$$r = \frac{\ln 2}{30}.$$
   The equation is
$$10 = 60\, e^{-\frac{\ln 2}{30} t}.$$
   So
$$t = \frac{\ln \frac{10}{60}}{-\frac{\ln 2}{30}} \approx 77.55 \text{ years}.$$

15. You have to solve $x^2 - 1 > 0$. The answer is $(-\infty, -1) \cup (1, \infty)$.

16. $g^{-1}(x) = \frac{3-3^x}{2}$.

17. [CALCULATOR] A person won a lottery ticket of \$500,000 and decided to put it in a saving account.

(a) $500,000 * e^{0.06*2} \approx \$563748.43$.

(b) The equation is

$$500,000 * e^{0.06*2} = 500,000 * \left(1 + \frac{0.06}{4}\right)^{4t}.$$

So

$$t = \frac{0.06 * 2}{4 \ln(1 + \frac{0.06}{4})} \approx 2.01 \text{ years}.$$

18. (a) Do as required!

(b) $-1.14,\ 0.92$.

(c) $[-2, -1.14) \cup (0.92, 2]$.

19. [CALCULATOR] The cat population in Tokyo has a relative growth rate of 1% per year. The estimated population in 2005 was $10,000$.

(a) $10,000 * e^{0.01*10} \approx 11052$.

(b) The equation is

$$20,000 = 10,000\, e^{0.01t}.$$

Hence

$$t = \frac{\ln \frac{20,000}{10,000}}{0.01} \approx 69 \text{ years}.$$

20. (a) $\{x : x \neq 1, x \neq 2\}$

(b) $\{x : x \leq 1 \text{ or } x \geq 2\}$.