Math 115: Precalculus
Practice test for the final exam

You will only need a calculator for those problems which are marked with [CALCULATOR].

1. Let U be the unit circle centered at the origin O, and A be a fixed point on the unit circle. Fill in the following blanks:

   (a) If B is another point on the unit circle and the length of the circular arc AB is $\pi/4$, then the central angle $\angle AOB$ is ____________ radians.

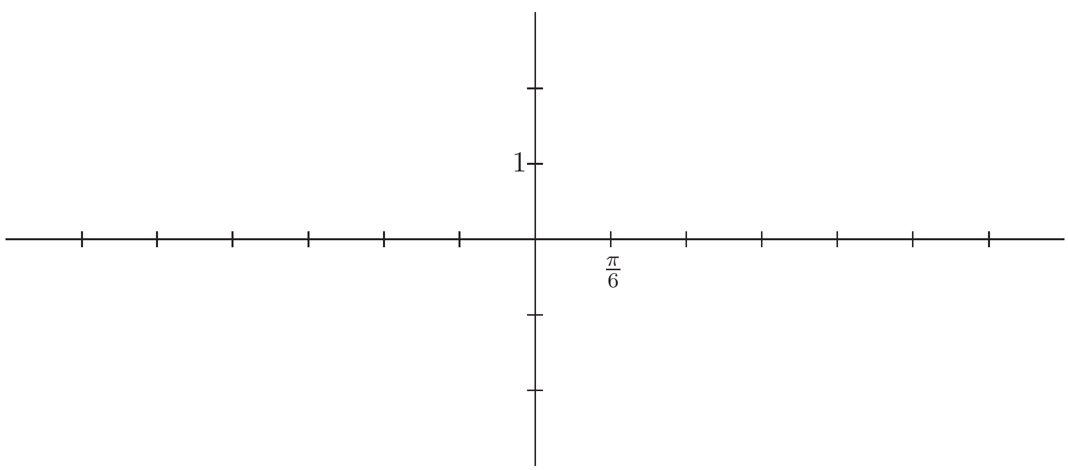
   (b) If B is as in (a) and the coordinate of A is $(1, 0)$, then the coordinate of B is ____________.

   (c) If B is as in (a), then $\sec(\angle AOB) =$ ____________.

2. Let $f(x) = -2\cos(3x - \pi)$.

   (a) Fill in the blank:
   - Amplitude = ________________________
   - Period = ________________________
   - Frequency = ________________________
   - Phase shift = ________________________

   (b) Sketch the graph of $f(x)$ in at least one complete period. Make sure that all the x-intercepts, maximum and minimum points, amplitude, period, and phase shift are labelled correctly.



3. Solve the following equation:

$$\log_2(x-3)+3=\log_2(2x).$$

4. (a) Find the exact value of $\cos\theta$ given that $\sin\theta=-0.32$ and $\cot\theta<0$.

   (b) Find the exact value of $\sin 2\theta$ and $\cos 2\theta$.

5. Let $g(x)=(2x+1)^3$. It is true that $g(x)$ is one-to-one. Fill in the following blanks:

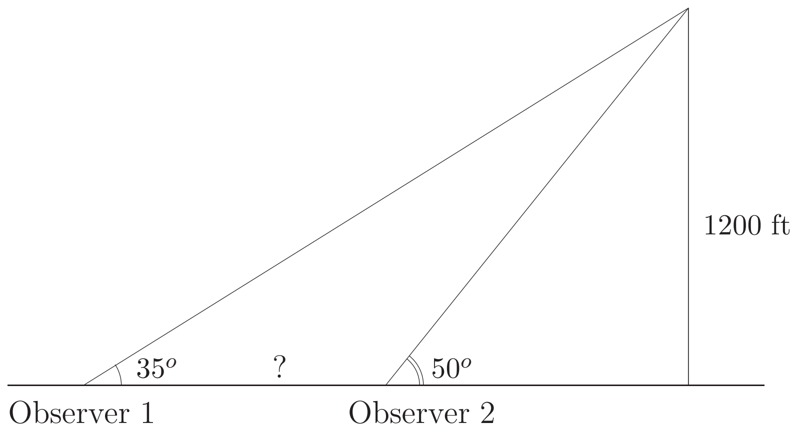
   (a) The graph of $g(x)$ intersects each vertical line at ____________________ one point.

   (b) The graph of $g(x)$ intersects each horizontal line at ____________________ one point.

   (c) Since $g(1)=27$ and g is one-to-one, $g^{-1}(27)=$ ______.

   (d) $g^{-1}(x)=$ ___________________________.

6. An observer stays on a level plain see the top of the mountain which is 1200 ft above the level plain at an angle of elevation of 35^o. Another observer standing at the base of the mountain sees the top at an angle of elevation of 50^o. (See the picture below.)



(a) Find the exact distance between the two observers.

(b) [CALCULATOR] Give an appoximate distance corrected to 4 decimal places: ______________.

7. [CALCULATOR] Two straight roads diverges at an angle of 70^o. At noon, a motocyclist leaves the intersection along one road at 24 mi/h. An hour after that, a car starts travelling on the other road at 40 mi/h. Find the distance of the car and the motorcycle at 3pm.

8. Solve the following equation in $[0, 2\pi)$ by following the subsequent steps.

$$2\cos^2 t - 3\cos t - 2 = 0.$$

(a) Solve the equation

$$2x^2 - 3x - 2 = 0.$$

(b) Among the solutions obtained in (a), which one lies in the range of cosine?

(c) [CALCULATOR] Solve the original equation by setting $\cos t$ equal to the solutions obtained in (b).

9. (a) Solve for the exact solution(s) of

$$2^{2x+3} = 5 * 3^{3x-2}.$$

(b) [CALCULATOR] Approximate the solution(s) up to 4 decimal places: ____________________.

10. [CALCULATOR]

   (a) Use a graphing calculator to draw the graphs of $f(x) = \log_2(2+x)$ and $g(x) = -3\sin(\pi - x)$ on the same set of axes. Sketch the graphs in the space provided below.

   (b) Approximate the solution of the equation $f(x) = g(x)$ corrected to 2 decimal places: ________________________.

   (c) Solve the inequality $f(x) > g(x)$. Pay attention to the domain of $f(x)$.

11. Find the inverse of $f(x) = e^{2x-1} + 3$.

12. Solve

$$2\,e^x - x^2\,e^x = 0.$$

   DO NOT approximate the solution(s).

13. Prove the following identity by carrying out subsequent steps:

$$\frac{\sec x + \csc x}{\tan x + \cot x} = \sqrt{2}\,\sin(x + \pi/4).$$

(a) Simplify the left hand side.

(b) Use addition formula to expand $\sin(x + \pi/4)$.

(c) Prove the identity.

14. The half-life of Cesium-137 is 30 years.

    (a) If the original sample weighs 60 grams, how much will remains after 60 years?

    (b) [CALCULATOR] After how long will only 10 grams of the 60-gram sample remain?

15. Find the domain of $\log_{3.14}(x^2 - 1)$. Give your answer in interval notation.

16. Find the inverse of $g(x) = \log_3(3 - 2x)$.

17. [CALCULATOR] A person won a lottery ticket of $\$500,000$ and decided to put it in a saving account.

    (a) Find the total sum after 2 years if the account pays 6% interest per year, compounded conitnuously.

    (b) If the interest rate remains the same but the sum is compounded quarterly instead, how long does it take to reach the same final sum?

18. [CALCULATOR] Let $f(x) = x^2 + 1$ and $g(x) = -x^3/5 + 2$.

(a) Graph $f(x)$ and $g(x)$ on the same set of axes with viewing rectangle $[-2, 2] \times [0, 5]$. Sketch the graph obtained in the space provided below.

(b) Solve the equation $f(x) = g(x)$ in the interval $[-2, 2]$ accurate to 2 decimal places. $x =$ ________________________.

(c) Solve the inequality $f(x) > g(x)$ in the interval $[-2, 2]$. Express your answer in interval notation.

19. [CALCULATOR] The cat population in Tokyo has a relative growth rate of 1% per year. The estimated population in 2005 was $10,000$.

(a) Find the projected cat population in 2015.

(b) How many years does it take for the population to reach $20,000$?

20. (a) Find the domain of

$$\frac{x^2 + 1}{x^2 - 3x + 2}.$$

(b) Find the domain of

$$\sqrt{x^2 - 3x + 2}.$$