Math 115: Precalculus Special review for the not-so-easy final exam: EXP and LOG

(1) Definitely, you have to look at the review for the second exam. Part of it is repeated here though not as complete.

(2) Problems 1-24, p. 349. These are the simplest possible problems about logarithmic function. If till now you are still not quite familiar with logarithm, they are your buddies to help you. Learn the definition of log, then try to do these problems by your bare eyes. If you need more than 30 seconds to do one problem, that's inadequate.

(3) Problems 13-38, p. 357. They are the second simplest type of problems in this review: Expanding and combining logarithms. Typically, 1-2 minutes for one problem is good.

(4) Problems 59-64, p. 351. These problems ask to find the domain of a logarithmic expression. For a logarithmic expression to be defined, the argument of every logarithmic term has to be positive, strictly positive. For example, the domain of $\log(x^2 - 3x + 2)$ is $\{x \in \mathbb{R} | x^2 - 3x + 2 > 0\}$. To find out what this set is, you need to solve the inequality $x^2 - 3x + 2 > 0$. For this, factor $x^2 - 3x + 2$ as $(x - 1)(x - 2)$. Now, do whatever you want, make a table of signs, checking different cases, or any other method you know, but make sure that you get the right thing, $x < 1$ or $x > 2$.

(5) Problems 25-38, p. 337, 49-58, p. 350. You need to do graphing in these problems. In addition, you need to find domain, range and asymptote. It should be noted that exponential functions have horizontal asymptotes while logarithm s have vertical asymptotes.

(6) Problems 1-50, pp. 366-367. I've done enough on this in the last review, so I'll not bore you by repeating it here.

(7) Problems 73-82, p. 340, 67-74, p. 367 (compound interest), 1-13, pp. 379-380 (population), 14-22, pp. 380-381 (radioactive decay). Have a look at the review for the second exam for more stuffs. Attention: Even though the formulas are given to you in the exam, you still need to learn them by yourselves prior to enter the exam. How meaningful is a formula without any specification about

which quantity is which? Last time, many of you were staring at the formula for radioactive decay having no sense of what should be plugged in.

Since I didn't do any sample population growth problems last time, let me work out one here, say problem 6, p. 380. More or less this is similar to continuous compound interest. What you need to keep track of are:

- Initial population
- Growth rate
- Time
- Final population

The formula is

$$\text{Final population} = \text{Initial population} \times e^{\text{Growth rate} \times \text{Time}}.$$

In this particular problem, the initial frog population is 85, growth rate is 18% per year. We need to find (a) the formula, (b) the population after a time of 3 years, and (c) time required for the frog population to reach 600.

(a) is straightforward:

$$n(t) = 85e^{.18t}$$

where $n(t)$ is the frog population after t years.

For (b), we apply the formula we just obtained:

$$n(3) = 85e^{.18\times 3} \approx 146(\text{frogs}).$$

For (c), we use the formula to write an equation:

$$600 = 85e^{.18t}.$$

To get time, we solve this equation:

$$\begin{aligned} e^{.18t} &= \frac{600}{85} \\ .18t &= \ln\frac{600}{85} \\ t &= \frac{\ln\frac{600}{85}}{.18} \approx 11(\text{years}). \end{aligned}$$