Math 115: Precalculus
Special review for the wacky final exam: Long ago, we did ...

1. Problems 37-56, pp. 109-110. You are asked to solve an equation or an inequality graphically. Amazingly, in the past exam, many of you seem to have problem with this type of problems. It shouldn't be that way, they are simple. My suggestion for you is to read the text and do these problems.

2. Problems 1-18, pp. 190-191. Transformations of graphs. What I'd like to say is: BEAT THEM. They're not hard, you just need to practice. Remember the order of the transformations does matter. What you need are: inside parentheses first, multiplication first, otherwise from left to right. I guess you know what I mean by these.

3. Problems 1-50, pp. 219-221. You definitely don't want to miss them in the exam. They are fairly easy. If they show up on the exam, I would call them "give-away-credit" problems.

4. Problems 17-50, pp. 230-231, 75-77, p. 351. Inverse functions are the main points of these problems. I bet this type of problems will for sure show up on the exam under some form.

   Here is the scheme: Say you want to find the inverse of $f(x)$. Set $y = f(x)$ and then try to solve for x in terms of y. Think of y as a real number.

   Let me do a few examples. The level of difficulty will increase as we go along. In all of the following example, the given functions are already one-to-one. (In particular, it's legitimate to talk about their inverses.)

   (a) Inverse function of a linear function: Find the inverse of $f(x) = 3x - 1$. Set $y = 3x - 1$. We want to express x in terms of y. As an equation of x, this is a linear equation, so we solve it by tossing the x-terms to one side, and non-x-terms to the other side and then proceed:

$$\begin{aligned} y &= 3x - 1, \\ 3x &= y + 1, \\ x &= \frac{y+1}{3}. \end{aligned}$$

   We conclude that the inverse function is $f^{-1}(y) = x = \frac{y+1}{3}$.

(b) Inverse function of a fractional function: Find the inverse function of $g(x) = \frac{x-1}{2x+3}$.

We do the same think, set $y = \frac{x-1}{2x+3}$ and solve for x in terms of y.

> Think about how you would solve the equation $1 = \frac{x-1}{2x+3}$. I believe you know what you should do in this case: Multiply both sides by $2x+3$, and then you end up with a linear equation of x.

We will do the same thing, no more, no less. It's just the appearance of y that makes the difference. Our method is to ignore ... some aspect of it. Here is how we attack it:

$$\begin{aligned} y &= \frac{x-1}{2x+3}, \\ y(2x+3) &= x-1, \\ (2y)x + 3y &= x-1, \\ (2y-1)x &= -3y-1 \qquad \text{\% tossing the } x\text{-terms ... and non-}x\text{-terms ...} \\ x &= \frac{-3y-1}{2y-1}. \end{aligned}$$

Okay, the inverse function is $g^{-1}(y) = x = \frac{-3y-1}{2y-1}$.

(c) ... $t(x) = \sqrt[3]{(x-2)}$.

The equation we want to solve is

$$y = \sqrt[3]{(x-2)}.$$

Recall that the action inverse to taking root is to raise to some power. With this in mind, the equation is fairly easy to solve:

$$\begin{aligned} y^3 &= x-2, \\ x &= y^3 + 2. \end{aligned}$$

The inverse function is $t^{-1}(y) = y^3 + 2$.

(d) ... $m(x) = -(x-1)^6$, $x < 1$.

We solve

$$y = -(x-1)^6.$$

This is easy, right? The equation is equivalent to $-y = (x-1)^6$. Taking the sixth root of both sides we have

$$\sqrt[6]{-y} = x-1.$$

So $x = \sqrt[6]{-y} + 1$.

Had you agreed with me up to this point, you'd have been totally WRONG! **WRONG!** During the course of taking root of even order, we have forgotten an important factor: the absolute value signs. The correct thing we should get is

$$\sqrt[6]{-y} = |x - 1|.$$

To delete the absolute signs, we need to check if the argument inside the absolute signs is positive or negative. As $x < 1$, $x - 1$ is negative, so $|x-1| = -(x-1) = -x+1$. So

$$\sqrt[6]{-y} = -x + 1$$

and $x = 1 - \sqrt[6]{-y}$. The inverse function is $m^{-1}(y) = 1 - \sqrt[6]{-y}$.

(e) ... $h(x) = 3\,e^{4x+2}$.

We are now solving

$$y = 3\,e^{4x+2}.$$

> Ask yourself how you solve $1 = 3\,e^{4x+2}$. HINT: Toss all the non-exponential terms to one side then apply the sibling of exponential functions, logarithms.

The magic trick (or treat) works here as well.

$$\begin{aligned} y &= 3\,e^{4x+2}, \\ \frac{y}{3} &= e^{4x+2}, \\ \ln\frac{y}{3} &= \ln e^{4x+2}, \\ \ln\frac{y}{3} &= 4x + 2 \\ \ln\frac{y}{3} - 2 &= 4x, \\ x &= \frac{1}{4}\big(\ln\frac{y}{3} - 2\big). \end{aligned}$$

The inverse function is then $h^{-1}(y) = \frac{1}{4}\big(\ln\frac{y}{3} - 2\big)$.

(f) ... $k(x) = 2\log_3(2x-3)$.

It should be familiar to you now:

$$\begin{aligned} y &= 2\log_3(2x-3), \\ \frac{y}{2} &= \log_3(2x-3). \end{aligned}$$

Now, we use exponential functions instead of logarithmic ones. We get

$$\begin{aligned} 3^{y/2} &= 3^{\log_3(2x-3)}, \\ 3^{y/2} &= 2x - 3, \\ 3^{y/2} + 3 &= 2x, \\ x &= \frac{1}{2}\big(3^{y/2} + 3\big). \end{aligned}$$

The required inverse function is $k^{-1}(y) = \frac{1}{2}\big(3^{y/2} + 3\big)$.

5. Problems 5-56, p. 313. Here come the painful, ugly, whatever, rational functions. Let's consider problem 42. We need to find all the intercepts and asymptotes of $s(x) = \frac{2x-4}{x^2+x-2}$. Before proceeding to the next step, please review how to find horizontal and vertical asymptotes of a rational functions.

   y-intercept is easy to find. I just need to set $x = 0$ and find $s(0)$:

   $$s(0) = \frac{2.0 - 4}{0^2 + 0 - 2} = 2.$$

   So the y-intercept is 2.

   For x-intercept(s), I need to solve $s(x) = 0$, i.e.

   $$\frac{2x - 4}{x^2 + x - 2} = 0.$$

   For a rational expression to be zero, I need the nominator to be zero and the denominator not. Setting the nominator equal to zero, we find $x = 2$. This doesn't annihilate the denominator, so it's an x-intercept.

   For vertical asymptote(s), I set the denominator to zero and solve for x:

   $$x^2 + x - 2 = 0.$$

   You can solve this by factoring or using the quadratic formula. All give $x = 1$ or $x = -2$ (two numbers). So the vertical asymptotes are $x = 1$ and $x = -2$ (two vertical lines).

   For horizontal asymptote, I first compare the degree of the nominator and the denominator. If that of the former is smaller, the horizontal asymptote is $y = 0$. If it is bigger, there is not horizontal asymptote. If the degrees are equal, the horizontal asymptote is $y = \square$, where $\square$ is the ratio of the leading coefficients of the nominator and the denominator. In our case, the degree of the nominator is smaller, so $y = 0$ is the horizontal asymptote.

   For a graph, consult your calculator.