

# Math 151: Calculus

## Quiz #1 (due Monday, Feb. 19)

1. Let

$$f(x) = \frac{2e^x + 1}{3e^x - 5}.$$

Find all intercepts, vertical and horizontal asymptotes.

2. Use the limit laws to find

$$L = \lim_{x \rightarrow -2} x^3.$$

Then find a  $\delta > 0$  such that if  $|x + 2| < \delta$ , then  $|x^3 - L| < 0.1$ . Justify your answers.

3. Let  $P(x) = 2x^3 - 4x + 1$ . Show that  $P(x)$  has a zero in  $(0, 1)$ . (Graphical proof is not allowed.)
4. Calculate the following limits.

(a)  $\lim_{x \rightarrow 2} \frac{x(x^{3/2} + 1)}{\sqrt{x-1} - \sqrt{2}}.$

(b)  $\lim_{x \rightarrow -\infty} \frac{x^3 + 2x - 100}{2x^6 + 7}.$

(c)  $\lim_{x \rightarrow +\infty} (\sqrt{x^2 + 3x - 2} - x).$

5. Let

$$f(x) = \begin{cases} x^2 & \text{if } x \leq -1 \\ A|x| - 1 & \text{if } -1 < x < 1 \\ x^3 - Bx + 1 & \text{if } x \geq 1. \end{cases}$$

- (a) For which values of  $A$  and  $B$  is  $f(x)$  everywhere continuous? Justify your answer.
- (b) With  $A$  and  $B$  found in (a), where does  $f(x)$  fail to be differentiable? Explain your answer.
6. Suppose that  $f(x)$  and  $g(x)$  are differentiable functions such that

$$f(4) = 2, \quad f'(4) = -1, \quad g(4) = 1, \quad g'(4) = -3.$$

Compute the derivatives of the following functions at  $x = 4$ :

$$A(x) = f(x) + g(x), \quad B(x) = f(x)g(x), \quad C(x) = \frac{f(x)}{g(x)}.$$

7. Calculate the derivatives of the following functions:

(a)  $f(x) = \frac{2x+1}{3x-2}$ .

(b)  $g(t) = \frac{e^t \cos t}{(e^t+1)(3-\sin t)}$ .

(c)  $h(y) = y^3 + \frac{\tan y}{1+y^2}$ .

8. Find the equation of the line that is tangent to the curve

$$y = \frac{x}{x-1}$$

at the point with  $x$ -coordinate 2.

9. Let

$$H(x) = x^2 + 3x + 1$$

(a) Calculate

$$\frac{H(a+h) - H(a)}{h}.$$

(b) Based on (a), find the limit

$$\lim_{h \rightarrow 0} \frac{H(a+h) - H(a)}{h}$$

wherever it exists.

(c) Calculate the derivative of  $H(x)$  and check if  $H'(a)$  is the same as the limit obtained in (b).

10. Find the derivative of

$$g(t) = \frac{2t^2 + t - 1}{t^2 - 2t - 3}.$$

Find all intercepts, vertical and horizontal asymptotes of  $g'(t)$ .