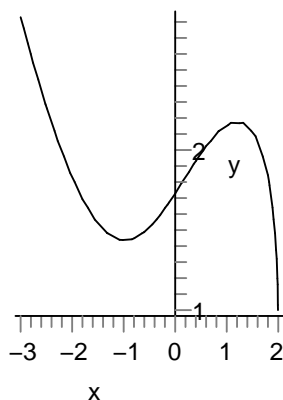


Math 151: Calculus

Quiz #2 (due Monday, Apr. 2)

1. Let $g(x) = \sqrt[5]{x-1}$. Write the linear approximate $L(x)$ for $g(x)$ near $x = 33$. Then use the linearization to approximate $\sqrt[5]{33}$.
2. A function $y = f(x)$ is defined implicitly by $x^3 - 2xy + 2y^2 = 6$, $y \geq 1$.



- (a) Find an expression for $f'(x)$ in terms of x and $f(x)$.
 - (b) Write the equation for the tangent line of the graph of f at the point whose x -coordinate is -1 .
 - (c) Find critical points of f , i.e. points x at which $f'(x)$ vanishes or does not exist. Distinguish between the two possibilities.
 - (d) What is the equation of the tangent line of the graph of f at the point $(2, 1)$?
3. Let $P(x) = x^4 - 3x^2 + 6x - 5$. Assume that we are using Newton's method to find root(s) of $P(x)$.
 - (a) Give a recursive formula expressing the $(n+1)$ -st approximation x_{n+1} in terms of the n -th approximation x_n .
 - (b) Start with $x_0 = -2$, find a root of $P(x)$ accurate to three decimal places.
4. Calculate the following limits:

<ol style="list-style-type: none"> (a) $\lim_{x \rightarrow 0} \frac{\sin^2 x}{x - x \cos x}$, (c) $\lim_{x \rightarrow \infty} (x^2 - 3x + 1)^{1/x}$, 	<ol style="list-style-type: none"> (b) $\lim_{x \rightarrow 0} (\csc x - \cot x)(\sec x - \tan x)$, (d) $\lim_{x \rightarrow \infty} (xe^{1/x} - x)(e^x + x)^{1/x}$
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5. Let $f(x) = \frac{2x+3}{x-2}$. Is there any number $c \in [0, 5]$ such that $f(5) - f(0) = 5f'(c)$? If yes, find the exact value of c . If no, why does this not contradict the mean value theorem?
6. Let $h(x) = x^3 - 3x^2 - 6x + 2$. Find the exact values of the following quantities:
- critical points of h ,
 - maximum and/or minimum value(s) of h in the interval $[-3, 4]$,
 - and points of relection of the graph of h .
7. Assume that f is a differentiable function on the whole real line. Suppose for some reason you know that
- $f(-1) = 2, f(0) = 3, f(2) = -4$,
 - $f'(-1) = f'(0) = f'(2) = 0$,
 - $f' < 0$ in $(-\infty, -1) \cup (0, 2)$ and $f' > 0$ in $(-1, 0) \cup (2, \infty)$.
 - $\lim_{x \rightarrow \pm\infty} f(x) = 4$.
- Find all relative maxima and minima of f .
 - Decide whether f has an absolute maximum and/or an absolute minimum. If so, what are they?
 - What are the intercepts of f ?
 - Does the graph of f have any asymptotes? If yes, what are their equations?
 - Assume that f is twice differentiable, i.e. f'' exists everywhere. Why does the graph of f have at least four reflection points?
 - Show that there exists some $c \in [-1, 0]$ such that $f'(c) = 1$. Can c be 0 or 1?
 - Sketch the graph of f . Make sure to show important points and lines.
8. Find $r(t)$ given that $r(0) = 3$ and $r'(t) = \sin t + t^3 - 2t + 1$.
9. A person pulls a boat into a dock by a rope attached to the bow of the boat and passing through a pulley of the dock that is $2m$ higher than the bow of the boat. How fast is the boat moving towards the dock when it is 10 m away from the dock? It's known that the person pulls the rope at a constant rate of 0.5 m/s.
10. A plank 16 ft long rests against a vertical wall. If the bottom of the plank is sliding away from the wall at a speed of 1 ft/s, how fast is the angle between the top of the plank and the wall changing when the angle is $\pi/3$?