

# Math 151: Calculus

## Workshop #1

1. Let  $f(x) = \sqrt{x}$  and  $g(x) = \log x$ . Find the domain and range of each of the following functions and justify your assertions.

$$F(x) = \frac{f(x)}{g(x)}, \quad G(x) = f(x) - g(x), \quad H(x) = f(x) + \frac{1}{g(x)},$$
$$K(x) = f(g(x)), \quad L(x) = g(f(x)).$$

2. A piece of wire 180 inches long is bent into the shape of an isosceles trapezoid whose base angles are  $\pi/3$  radians.



- (a) Let  $x$  be the length of the longer base of the trapezoid and  $y$  the length of one of the slanted sides. Label the lengths of all sides in terms of  $x$  and  $y$  and deduce a relationship between  $x$  and  $y$ . HINT: What is the height of the trapezoid? How much do the two bases differ from each other?
- (b) Find a formula for the area  $A$  of the trapezoid as a function of the single variable  $x$ .
- (c) Use your calculator to graph the function  $A = A(x)$ . Are there any upper or lower bounds between which the value of  $x$  must lie? If so, decide what happens to  $A$  as  $x$  approaches those limits, and explain by drawing pictures of the trapezoid in those cases.
3. (a) Graph  $y = |x - 2|$  and  $y = |x - 3|$  on the same set of axes. From this graph, decide for which real numbers  $x$  the inequality  $|x - 2| < |x - 3|$  is valid.
- (b) Interpreting  $|a - b|$  as the distance between  $a$  and  $b$  on the number line, give a geometrical explanation why your solution in (a) is correct.
- (c) Repeat for the inequality  $|x + 1| < 2|x - 1|$ .
- (d) The inequality  $x + 1 < 2(x - 1)$  has a different solution (what is it?) from the inequality in (c). Explain algebraically why these two inequalities cannot be expected to have the same solutions.

4. A tasty bug is at point  $(x, 0)$  on the  $x$ -axis and is being watched by a sparrow perched at the point  $(4, 9)$  and a woodpecker who is at the point  $(-2, 5)$ .
- Find a formula for the distance from the sparrow to the bug, and a formula for the distance from the woodpecker to the bug.
  - For what values of  $x$  is the bug nearer the sparrow? Give an algebraic solution and a graphical solution.
5. For each of the following equations, (a) say how many solutions there are in the interval  $0 \leq x \leq \pi$ ; (b) find them all numerically, using a graph of the function  $f(x) = 4^{\sin 2x}$ , (c) obtain exact expressions for all the solutions satisfying  $0 \leq x \leq \pi$ , in terms of the log and arcsin (or  $\sin^{-1}$ ) functions, and constants such as  $\pi$ ; (d) check on the calculator that your answers to (b) and (c) coincide.

$$2 \cdot 4^{\sin 2x} = 1, \quad 3 \cdot 4^{\sin 2x} = 1.$$

6. An employer presently pays each of his workers \$10.00 per hour plus an additional piecewise rate of \$1.20 per unit produced each hour. In order to increase productivity to at least 4 units per hour for each worker, he decides to lower the hourly pay to \$8.40 and increase the piecewise rate.
- Find a linear equation for the (total) hourly wages  $W$  which are presently paid to each worker, in terms of the number  $x$  of units presently produced per hour by each worker. What is the slope of the line? What is the  $W$ -intercept?
  - What new piecewise rate should the employer pay so that workers in the new set-up who produce 4 units per hour would make the same total wage as in the old set-up, again assuming a linear relation between  $W$  and  $x$ ? What is the formula for  $W$  in the new set-up?
  - If you were an employee and wanted to increase your wages, would your employer's strategy induce you to produce more units? Explain your answer.