

# Math 151: Calculus

## Workshop #2

1. Let

$$f(x) = \begin{cases} 2x^2 + 2, & \text{if } x < 1 \\ ax^2 + bx, & \text{if } 1 \leq x \leq 2 \\ 10 - \frac{2}{x}, & \text{if } x > 2 \end{cases} .$$

- (a) Suppose that  $a = 2$  and  $b = -3$ . Graph  $f(x)$  for  $0 \leq x \leq 3$ . Find the left and right hand limits of  $f(x)$  as  $x$  approaches 1 and as  $x$  approaches 2.
- (b) Find  $a$  and  $b$  so that the graph of  $f(x)$  doesn't have any jumps.
- (c) For the  $a$  and  $b$  which you found in (b), find the slopes of the tangent lines to  $2x^2 + 2$  and to  $ax^2 + bx$  at  $x = 1$ . What do your results tell you about the graph of  $f$  at  $x = 1$ ?
- (d) Do a similar calculation for the graph at  $x = 2$ .

2. Let  $f(x) = \sin x - \frac{3x}{2}$ .

- (a) Use a calculator to sketch the graph of  $f$ .
- (b) Explain how the graph of  $f$  indicates that  $f$  is one-to-one.
- (c) Sketch the graph of the inverse function,  $g$ , of  $f$ .
- (d) Find the approximate value of  $g(4)$ .
- (e) Explain why the function  $\sin x - \frac{x}{4}$  is not one-to-one.

3. (a) Graph the function  $y = 3^x$  in the window  $-1 \leq x \leq 1$ . On the same axes draw the secant lines through the points  $(x, 3^x)$  and  $(0, 1)$  for  $x = 0.5$  and  $x = 0.1$ .
- (b) Let  $m(x) = (3^x - 1)/x$  be the slope of the secant line to the graph in (a). Graph  $m(x)$  for  $-1 \leq x < 0$  and  $0 < x \leq 1$ . Give a table of values of  $m(x)$  for  $x = -.02, -.01, .01, .02$ . (Use the TABLE key for the TI-82 or the EVAL key for the TI-85.)
- (c) Determine the limiting value of  $m(x)$  as  $x$  approaches 0, correct to five decimal places, by using a smaller  $\Delta T$  in the table setup. Later in the term we will show that the limit is  $\ln 3$ . Check this numerically.
- (d) Draw the secant line through  $(x, 3^x)$  and  $(0, 1)$  for  $x = 0.0001$ .
- (e) What is the equation of the tangent line to  $y = 3^x$  at  $(0, 1)$ ?

4. After a softball game, the winning pitcher throws the ball straight up in the air. The height  $s$  of the ball in feet is given by the formula  $s = 5 + 48t - 16t^2$ , where  $t$  is the time after release (measured in seconds).
- The formula is valid only until the ball hits the ground. When does that happen?
  - Find the instantaneous velocity of the ball at the following instants:  $t = 1$ ,  $t = 2$  and  $t = 3$ .
  - When do you think that the ball reaches its highest point? Check your guess by computing the instantaneous velocity at that instant. How high does the ball get?
5. An airplane is flying horizontally in a straight line and will pass directly over Newark airport. The pilot contacts a flight controller at Newark airport and is told that the line of “sight” between the airport and the plane makes an angle of 18 degrees with the horizon. The plane continues on course for 6 more miles, not yet passing over the airport, and is then told by the controller that the angle has changed to 32 degrees.
- Make a sketch showing all the information given.
  - How high is the airplane flying, and what was the straight-line distance from the plane to the airport when the plane initially contacted the controller? Give calculations to explain your answer.
6. Use your calculator to try to figure out what might be the value of

$$\lim_{x \rightarrow 0} \frac{(\cot x)(1 - \cos 2x)}{x}$$

by tracing the graph of  $(\cot x)(1 - \cos 2x)/x$  for  $x$  near 0. Show the graph. Then find the exact value of the limit by using a computation based on trig formulas and the fact that  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$ .

7. Let  $f_n(x) = (x^n)2^{-x}$ .
- Find the graphs of  $f_n(x)$  for  $0 \leq x \leq 10$  and  $n = 1, 2, 3$ . You will have to adjust the viewing window to see the graph. Describe how the graphs change as  $n$  increases. What features stay the same? Find the  $x$  coordinate  $x_{\max}$  of the highest point of the graph for  $n = 1, 2, 3$ . Plot  $x_{\max}$  as a function of  $n$ .
  - Guess what the graph of  $f_5(x)$  looks like, and what the  $x$  coordinate of the highest point is, based on the evidence from (a) and (b). Then test your guess by actually generating the graph.