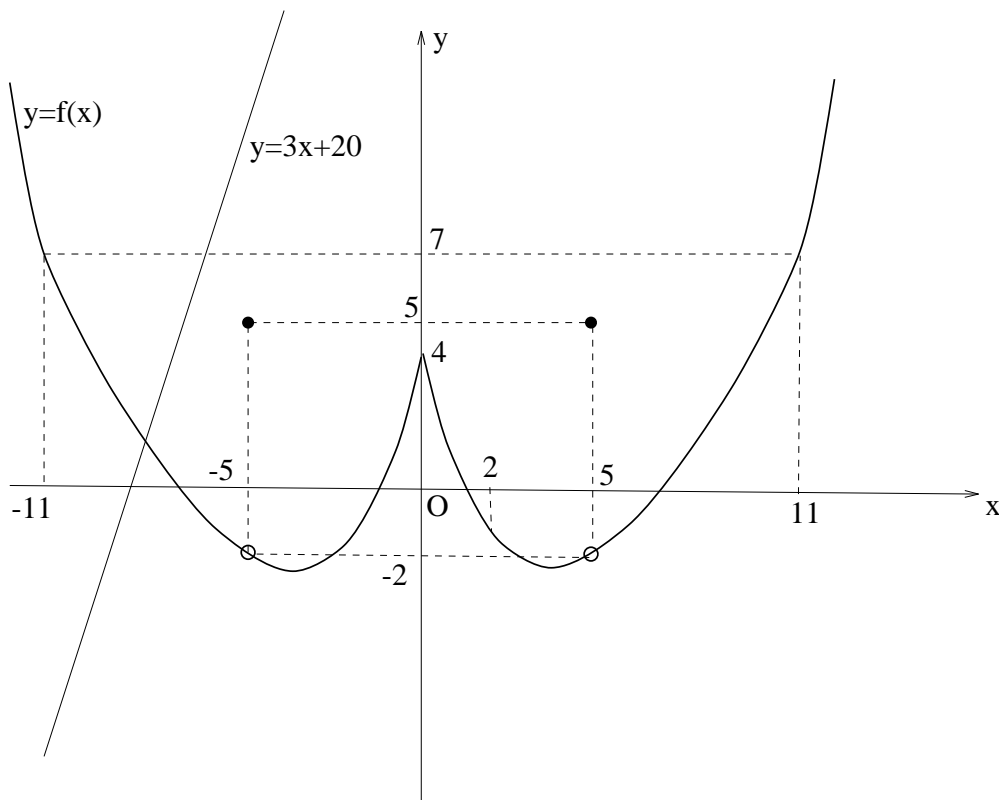


# Math 151: Calculus

## Workshop #4

- Let  $f(x) = x - \sqrt{x^2 + 8x + 1}$ .
  - Graph  $f(x)$  for  $1 \leq x \leq 20$ .
  - Use the TABLE key (or FEVAL on the TI-85) to investigate  $f(x)$  for large  $x$  (set TblMin= 1000 and  $\Delta\text{Tbl} = 1000$ ). Do you think that  $\lim_{x \rightarrow +\infty} f(x)$  exists? If you think the limit exists as a finite number, what is it?
  - Use algebra and limit theorems to prove your guess from (b).
- Determine whether each of the following statements is true or false (i.e. true in all examples, or false in at least one example). If you claim that the statement is true, then justify your claim by an appropriate limit law. If you claim that the statement is false, then justify your claim by giving an example.
  - If  $\lim_{x \rightarrow +\infty} f(x) = +\infty$  and  $\lim_{x \rightarrow +\infty} g(x) = -\infty$ , then  $\lim_{x \rightarrow +\infty} (f(x) + g(x)) = 0$ .
  - If  $\lim_{x \rightarrow 1} f(x) = 0$  and  $g$  is any function defined for all  $x$ , then  $\lim_{x \rightarrow 1} f(x)g(x) = 0$ .
  - If  $f(x) < 2$  for all  $x \neq 0$  and  $\lim_{x \rightarrow 0} f(x)$  exists, then  $\lim_{x \rightarrow 0} f(x) < 2$ .
- Let  $f(x) = x^2 + c$  and  $g(x) = 3x$ .
  - Plot the graphs of  $f(x)$  and  $g(x)$  for  $0 \leq x \leq 3$  for various values of  $c$ . Use the picture to get an approximate value of  $c$  for which the graphs of  $f(x)$  and  $g(x)$  are tangent to each other. In the subsequent steps, we will try to get this number algebraically.
  - Assume that the graphs of  $f(x)$  and  $g(x)$  are tangent to each other at some point  $P$ . Find a relationship between the slope of the tangent lines to the graphs of  $f(x)$  and  $g(x)$  at  $P$ .
  - Use the definition of derivatives, prove that  $f'(x) = 2x$  and  $g'(x) = 3$ .
  - If the point  $P$  has coordinate  $(a, b)$ , use (b) and (c) to find the value of  $a$ .
  - Use the fact that  $P$  lies at the same time on the graph of  $f(x)$  and on the graph of  $g(x)$ , find the value of  $b$  and then of  $c$ . Check if your answer in (a) is good or not.

4. At time  $t = 0$ , Irwin fires a rubber band at a spider on the ceiling. At time  $t = 3$  it hits the mark; then it falls. Let  $s(t)$  represent the height of the rubber band above the floor at time  $t$ .
- Which is larger,  $s(1.3)$  or  $s(1.7)$ ?
  - Which is larger,  $s'(1.3)$  or  $s'(1.7)$ ?
  - Sketch possible graphs of  $s$  and  $s'$  for  $0 \leq t \leq 4$ .
5. Consider the function  $y = f(x)$  whose graph is given in the figure below. The function  $f(x)$  is even. The tangent line to the graph  $y = f(x)$  at  $x = 11$  is parallel to the line  $y = 3x + 20$ .



- Is  $f(x)$  continuous at  $x = 0$ ? Is it differentiable at  $x = 0$ ? Explain why.
- Is  $f(x)$  continuous at  $x = 5$ ? Is it differentiable at  $x = 5$ ? Explain why.
- Is it true that  $f'(2) > 0$ ? Is it true that there is  $x \in (0, 5)$  such that  $f'(x) = 0$ ? Explain why.
- Find an equation of the tangent line to the graph  $y = f(x)$  at  $x = -11$ . Explain how the answer was obtained.