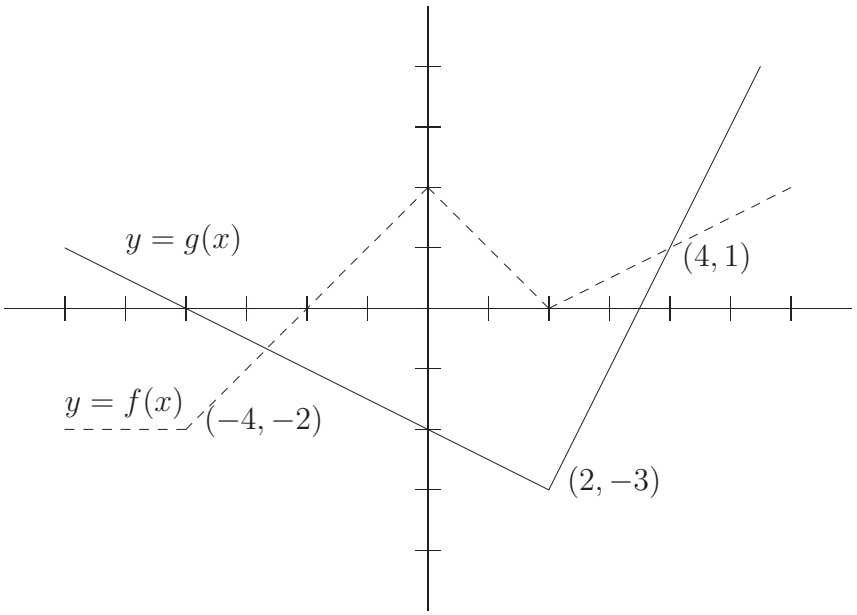
Math 151: Calculus
Workshop #6

1. From the point $(8, 0)$ you can draw tangent lines to the parabola $y = x^2 - x$. How many such tangent lines are there, and where are the points of tangency? Graph the parabola and the tangent lines.

2. The graphs of two functions f and g are shown in the following figure:



   (a) Suppose that $F(x) = f(x) - 3g(x)$. Find $F(-2)$ and $F'(-2)$.

   (b) Suppose that $G(x) = f(x)g(x)$. Find $G(0)$ and $G(3)$. Determine whether or not $G'(0)$ and $G'(3)$ exist and find the values if they exist.

   (c) Suppose that $H(x) = f(x)/g(x)$. At what points x is H discontinuous? At what points x is H not differentiable? Explain.

3. Suppose $f(x)$ is a differentiable function. Using the differentiation rules we have studied so far, find expressions for the derivatives of the following functions in terms of $f(x)$ and $f'(x)$:

$$\text{(a) } f(x)e^x, \qquad \text{(b) } \frac{2 + f(x)}{3 - f(x)}, \qquad \text{(c) } f(x)^2, \qquad \text{(d) } \frac{x^3}{f(x)}.$$

4. Consider the curve defined by the the equation $-8x^2 + 5xy + y^3 = -149$.

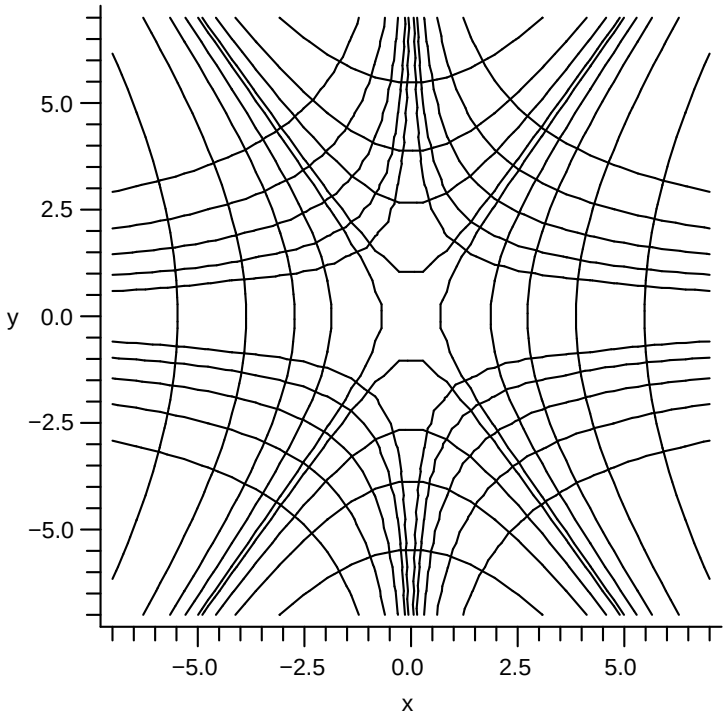
   (a) Compute dy/dx.

   (b) Write an equation for the tangent line to the curve at the point $(4, -1)$.

   (c) There is a number k such that the point $(4.2, k)$ is on the curve. Using the tangent line found in (b), approximate the value of k.

5. In this problem you will consider two families of curves. The first family consists of the curve $xy^2 = 1$, the curve $xy^2 = 2$, etc., that is, the curves $xy^2 = C$, one for each constant C. The second family consists of the curves $2x^2 - y^2 = C$, again one for each constant C.

   (a) Identify each family in the following picture:



   (b) Show that any curve in the first family and any curve in the second family are perpendicular wherever they happen to meet. *For this reason the two families are said to be "orthogonal". Orthogonal families play a very important role in the study of electromagnetism and heat flow. Besides, they make elegant pictures.*