

Math 151: Calculus

Workshop #7

- An unidentified object moves along the s -axis, with displacement $s = s(t)$ (meters), velocity $v = v(t)$ (m/s) and acceleration $a = a(t)$ (m/s²). It so happens that the velocity and displacement are related by the equation $v = \sqrt{8s + 16}$. Moreover, at the instant $t = 0$, the object is observed at $s = 6$.
 - Show that a is constant, and find its value.
 - Based on (a), can you guess what the formula for v in terms of t is? Give reasoning for your guess.
 - Graph v as a function of s and then graph v as a function of t .
- Find the following derivatives:
 - $\frac{d}{dx} ([\ln(1 + x^2)] [\ln(1 - x^2)])$,
 - $\frac{d}{dx} (\ln [(1 + x^2)(1 - x^2)])$,
 - and $\frac{d}{dx} \left(\frac{x^x \sqrt{1 + e^2}}{e^{2x} \sqrt{1 + x^2}} \right)$.
- An object is moving along the parabola $y = 3x^2$.
 - When it passes through the point $(2, 12)$, its “horizontal” velocity is $\frac{dx}{dt} = 3$. What is its “vertical” velocity at that instant?
 - If it travels in such a way that $\frac{dx}{dt} = 3$ for all t , then what happens to $\frac{dy}{dt}$ as $t \rightarrow +\infty$?
 - If, however, it travels in such a way that $\frac{dy}{dt}$ remains constant, then what happens to $\frac{dx}{dt}$ as $t \rightarrow +\infty$?
- A worker pulls a 16 foot plank up the side of a building at 0.5 feet per second by means of a rope attached to one end of the plank. Assume that the top of the plank moves up the vertical side of the building, and that the bottom of the plank moves along level ground towards the wall. How fast is the bottom of the plank moving at the moment that the top is 8 feet above ground?
- An airplane is flying straight and level at an altitude of 6 miles. At noon it passes directly over a radar station. Some time later the radar operator sees that the (straight-line) distance between the plane and the station is 10 miles and is increasing at 240 miles per hour. What is the speed of the plane at that moment?

6. Imagine you are zooming in on the graph of each of the following functions *near the origin*. Group together the functions that become indistinguishable (you should get two distinct groups). Give reasons for your groupings. (Hint: the graphs of the functions in each group resemble the graph of a particular straight line when you zoom in far enough; explain this in terms of derivatives.) Write down the equation of the tangent line at the origin for each group.

$$f_1(x) = \ln(x + 1), \quad f_2(x) = x^2, \quad f_3(x) = x \sin x,$$
$$f_4(x) = \frac{1}{2} \ln(x^2 + 1), \quad f_5(x) = x, \quad f_6(x) = \sin[\ln(1 + x)].$$

7. A spherical balloon is being inflated with hot air. At time $t = 5$ seconds the radius of the balloon is equal to 10cm and is increasing at the instantaneous rate of 3 cm/sec. Use differentials to find an approximate value of the volume of the balloon at time $t = 5.1$ seconds.