

Oral Qualifying Exam Syllabus

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Combinatorics and Experimental Mathematics

- **Basic Enumeration:** counting arguments, generating functions, recurrence relations, inclusion-exclusion, pigeonhole principle, Stirling numbers, Bell numbers, Catalan numbers, Eulerian numbers
- **Permutations and Tableaux:** Descents, Cauchy's Convolution Formula, Major index, Inversions, Applications of Inversions to Determinants and Graphs, Inversions in Multiset Permutations, Gaussian coefficients, the Füredi-Hajnal Conjecture, the Marcus-Tardos Theorem (Stanley-Wilf Conjecture), notion of pattern avoidance and results for patterns of length 3 and 4, Robinson-Schensted-Knuth Correspondence, Standard Young Tableaux, the Hook Length Formula, generating trees, posets on S_n , Bruhat order, weak Bruhat order, posets on pattern avoiding permutations, simplicial complexes of restricted permutations
- **Ramsey Theory:** Ramsey's Theorem, probabilistic lower bounds, van der Waerden numbers, $R(3, n)$ upper and lower bounds
- **Experimental Math:** Maple programming, Golay code, gambler's ruin, Gaussian Quadrature, sorting algorithms, Gray code, Cassini identities, Prüfer bijection
- **Hypergeometric functions and identities:** definition and representation, Sister Celine's Algorithm, The Fundamental Theorem of Hypergeometric Series, Gosper's Algorithm, similarity among hypergeometric terms, existence of telescoped recurrences, Zeilberger's Algorithm, the WZ method and dual identities.

Combinatorial Commutative Algebra

- **Monomial ideals:** degree, Hilbert function, Hilbert polynomial, radical, prime, irreducible, primary, primary decomposition

- **Gröbner bases:** monomial orderings, division algorithm, Dickson's lemma, Hilbert basis theorem, Buchberger's algorithm, Gröbner bases, minimal Gröbner bases, reduced Gröbner bases, properties of/applications of Gröbner bases, syzygies, improvements to Buchberger's algorithm, Hilbert Syzygy Theorem, Schreyer's algorithm for computing free resolutions
- **Other:** Macaulay's Theorem, Kruskal-Katona Theorem, Clements-Lindstrom Theorem