Errata for *The Classification of the Finite Simple Groups*, A.M.S. Surveys and Monographs 40

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ERRATA FOR NUMBER 1

These corrections to Number 1 have been included in the second printing.

Pages 47,140,142: *The correct Background Reference is:*
[**Ca1**] R. W. Carter, *Simple Groups of Lie Type*, Wiley and Sons, London, 1972.

Page 142: *The correct Expository Reference is:*
[**Ca2**] R. W. Carter, *Finite Groups of Lie Type: Conjugacy Classes and Complex Characters*, Wiley–Interscience, London, 1985.

Pages 100, 102: *In Definitions 12.1 and 13.1, the group $G_2(8)$ should be removed from the set $\mathcal{C}_3$ and placed in $\mathcal{T}_3$.*

ERRATA FOR NUMBER 2

Page 115, Line 6: ~~$J \cong SL_n(r^m)$, r odd~~ $J \cong SL_n(r^m)$, n and r odd

Page 117, Line -3: ~~$A \le C_{P^g}(u)$~~ $R_1 \le C_{P^g}(u)$

Page 117, Line -2: ~~$1 \ne R_1 \le A \le P \cap P^g \cap Y$~~ $1 \ne R_1 \le P \cap P^g \cap Y$

Page 122, Line 19: *In Definition 21.1* ~~p'-subgroups~~ A-invariant p'-subgroups

Page 172, Line -17: *In Lemma 29.5, a hypothesis needs to be added. The following is adequate, following the first sentence:* Assume that there is a mapping $\phi : E \to D$ such that $\phi(i) \ge i$ for all $i \in E$, and whenever $i, j \in E$ with $i \le j$, then $\phi(i) \le \phi(j)$.

ERRATA FOR NUMBER 3

Page 55, Line -11: *In the statement of Lemma 2.5.7:*
~~$C_{\mathrm{Aut}_1(K)}(K) = \langle \sigma \rangle$~~ $C_{\mathrm{Aut}_1(\overline{K})}(K, \sigma) = \langle \sigma \rangle$

Page 57, Line 18: *At the end of Definition 2.5.10, add:*
(f) $\mathrm{Aut}_0(K) =$ image of $C_{\mathrm{Aut}_0(\overline{K})}(\sigma)$ in $\mathrm{Aut}(K)$.

Page 58, Line -10: ~~If $K \cong A_m(q)$, $D_{2m+1}(q)$~~ If $K \cong A_m(q)$ $(m > 1)$, $D_{2m+1}(q)$

Page 261, Line -11: ~~Fi'_{24}~~ Fi_{23}

Page 275, Line 5 of "SMALL REPRESENTATIONS": ~~$\boldsymbol{F}_{5^2}$~~ $\boldsymbol{F}_{3^2}$

Page 279, Line -7: ~~$L_2(25)$~~ $L_2(25)\#2$

Page 288, Line -11: ~~$|M^{\#}|$~~ $|O_2(M)^{\#}|$

Page 290, Line 1: ~~$E(C(2A))$~~ $E(C(2B))$

Page 297, Line -18: ~~$K = Co_1$~~ $K = Co_0$

Page 299, Line 16: ~~lower bound for $P \times Q_8$ is 30~~ lower bound for a faithful complex representation of $P \times Q_8$ in which the involution of $Z(Q_8)$ acts as $-I$ is 30

Page 302, Line 12: ~~$E(C_K(z))$~~ $E(C_K(z_A))$

Page 302, Line 16: ~~becuase~~ because

Page 302, Line 19: ~~$|H|_3$~~ $|\overline{H}|_3$

Page 304, Line -13: ~~I is a homogeneous I-module~~ V_0 is a homogeneous I-module

Page 304, Line -8: ~~$z \in Q'_0$~~ $Z(J) \leq Q'_0$

Page 308, Line -17: ~~$B/Z(B) \cong 2\,{}^2E_6(2)$~~ $BZ(B) \cong {}^2E_6(2)//$

Page 309, Line -6: ~~$K \in \mathcal{K}$~~ $K \in \mathcal{K}$ and K is simple

Page 309, Line -1: ~~or $K \cong J_1$~~ or $K \cong {}^2G_2(3^{\frac{n}{2}})$, n odd, $n > 1$, or $K \cong J_1$

Page 314, Line 13: ~~$\dim(W_1)$~~ $\dim(W_i)$

Page 316, Line -2: ~~$V_q\alpha + V_q\beta$~~ $\boldsymbol{F}_q\alpha + \boldsymbol{F}_q\beta$

Page 316, Line -2: ~~$V_q\alpha$~~ $\boldsymbol{F}_q\alpha$

Page 317, Line 11: ~~$C_{(2\times 2)D_4(2)}(x)$~~ $C_{(2\times 2)D_4(2)}(x_1)$

Page 317, Line 16: ~~$|Sp_6(2)|$~~ $|Sp_6(2)|_2$

Page 319, Line 12: ~~Q is abelian~~ $\hat{Q}$ is abelian

Page 319, Line 13: ~~Q~~ $\hat{Q}$ *(twice)*

Page 319, Line 14: ~~Q is abelian~~ $\hat{Q}$ is abelian

Page 319, Line 15: ~~Q~~ $\hat{Q}$ *(four times)*

Page 329, Line 12: ~~$(r^{2a} + \epsilon r^a + 1)/3$~~ $(r^{2a} + \epsilon r^a + 1)/d$

Page 332, Line -3: *Theorem 6.5.5a misstates the structure of Borel subgroups of ${}^2G_2(3^{\frac{n}{2}})$, $n > 1$. The assertion should be:*

(a) Borel subgroups of K, of order $q^3(q-1)$. If $B = UH$ is such a Borel subgroup, with $|U| = q^3$ and $|H| = q - 1$, and if t is the involution of H, then $|C_U(t)| = q$, and the groups $O^2(B)$, $Z(U)H$ and $B/Z_2(U)$ are all Frobenius groups.

Page 333, Line -9: ~~$Z_2 \times L_2(q^2)$~~ $Z_2 \times L_2(q)$

Page 338, Line 1: *Replace this line by:* We proceed in a sequence of lemmas.

Page 338, Line 10: *Replace this line by:* We set $Y = K_1X$, so that $X \leq O_{r'}(Y)$, and next prove:

Page 345, Line 11: ~~$\Gamma_{E_2,*-1}(K)$~~ $\Gamma'_{E_2,*-1}(K)$

Page 345, Line 12: ~~$\Gamma_{E_2,*-1}(U) \le \Gamma_{E_2,*-1}(K)$~~ $\Gamma'_{E_2,*-1}(U) \le \Gamma'_{E_2,*-1}(K)$

Page 345, Line 13: ~~$\Gamma_{E_2,*-1}(K)$~~ $\Gamma'_{E_2,*-1}(K)$

Page 354, Line 14: *In the proof of Theorem 7.3.3, we omitted here a reduction to the case that $m_p(E) = 2$. This reduction is needed to justify the assertion in line 15 that $\Gamma = \Gamma_{E,*-1}(K)$. Thus, the following paragraph should be inserted before "We set":*

We first reduce the proof to the case $m_p(E) = 2$. Indeed, if the theorem holds in that case, then to complete the proof we must argue that if a noncyclic elementary abelian p-group E acts faithfully on K in such a way that one of the conclusions of 7.3.3 is satisfied by each $F \in \mathcal{E}_2(E)$, then E itself satisfies that same conclusion. This is accomplished by a few observations in the various cases. In case 7.3.3c, $\mathrm{Out}(K)$ has order 3 by 2.5.12, so $m_2(\mathrm{Aut}(K)) = m_2(K) = 3$ and the desired conclusion is obvious. In cases 7.3.3ehijkl, as well as the case $K = {}^2A_2(2)$ of 7.3.3a, it is immediate from 4.10.3 and 2.5.12 that $m_p(\mathrm{Aut}(K)) = 2$, with $\mathrm{Out}(K)$ a p'-group in case (e) and $m_p(K) = 1$ in cases (h) and (i). Thus the desired conclusions hold in these cases as well. In the remaining cases, it suffices to assume that $m_p(E) = 3$ and derive a contradiction. In cases 7.3.3df, $\mathrm{Out}(K)$ is a p'-group by 2.5.12, and 4.10.3ae implies that $m_p(K) = 3$ and that every element of K of order p lies in a conjugate of E. But in these cases of 7.3.3 it is stipulated that certain conjugacy classes of K of order p do not meet E, contradiction. In cases 7.3.3bg, we consider the character of E on the natural K-module, which (since $p \neq r$) lifts to a complex character χ. The conditions of cases (b) and (g) force $\chi(x) = -1$ for each $x \in E^{\#}$. As $(\chi, 1_E)$ is an integer, $\chi(1) \equiv -1 \mod p^3$. However, $\chi(1) = 5$ or 8, with $p = 2$ or 3, respectively, a contradiction. Finally, the only remaining case is that 7.3.3a holds and E acts on $K \cong L_p^{\epsilon}(q)$ like a subgroup $E^* \le GL_p^{\epsilon}(q)$, and the preimage F^* in E^* of any $F \in \mathcal{E}_2(E)$ satisfies $(F^*)' = \Omega_1(Z(K))$. But then $(E^*)' = \Omega_1(Z(K))$, and so $C_{E^*}(x)$ is a maximal subgroup of E^* for all $x \in E^* - Z(E^*)$. Choosing such an element x and using $m_p(E) = 3$, we find $y \in E^*$ such that $\langle x, y \rangle$ is abelian and has a noncyclic image in E, a final contradiction accomplishing our reduction.

Page 354, Line 22: ~~p'-subgroup~~ p'-subgroup

Page 357, Line -8: ~~its Lie components.~~ its Lie components. (See also Definitions 4.2.2 and 4.9.3, and Proposition 4.9.4.)

Page 358, Line -2: ~~${}^2F_4(2^{\frac{1}{2}})$~~ ${}^2F_4(2^{\frac{1}{2}})'$

Page 364, Line -15: ~~Then~~ Then if we define $\Gamma^{r'}_{\hat{E},*-1}(\hat{K})$ to be the subgroup of $\hat{K}$ generated by all r-elements centralizing some subgroup of $\hat{E}$ of index 2, we have

Page 365, Line -16: ~~$t_2^{(3)}$ and $t_2^{(4)}$~~ t_2'' and t_2'''

Page 381, Line -4: ~~Ψ_{ij}~~ Ω_{ij}

Page 381, Line -3: ~~$\Psi_{ij} \cup \Omega_0$~~ $\Omega_{ij} \cup \Omega_0$

Page 382, Line 3: ~~$\Gamma_{E,*-r}(K)$~~ $\Gamma'_{E,*-r}(K)$

Page 382, Line 5: ~~$A_{\Psi_{ij}}$~~ $A_{\Phi_{ij}}$

Page 382, Line 8: ~~then $\Psi_{ij} = \Phi_{ij}$;~~ then

Page 382, Line 9: ~~$4 = |\Phi_{ij}| = |\Psi_{ij}| = |\Omega_{ij}| + |\Omega_0|$~~ $4 = |\Phi_{ij}| = |\Omega_{ij}| + |\Omega_0|$

Page 382, Line 15: ~~$O^2(A_{\Phi_{ij}})$~~ $O^2(A_{\Phi_{ij}})$

Page 384, Line 1: ~~$SL_2(5) = 2A_1(4),$~~ $SL_2(5) = 2A_1(4)$, $(2)^2B_2(2^{\frac{3}{2}})$,

Page 396, Line -11: ~~K locally k-balanced~~ K is locally k-balanced

Page 399, Line -15: ~~irreducibly on P~~ irreducibly on $\Omega_1(P)$

Page 402, Line -4: ~~Theorem 7.8.1~~ Proposition 7.8.1

ERRATA FOR NUMBER 4

ERRATA FOR NUMBER 5

Page 3, Line -14: *In the definition of* $\mathcal{K}^{(7)*}$, *second line*
~~$\{A_4^\epsilon(q) \,|\, \epsilon = 1$ or q odd$\}$~~ $\{A_4^\epsilon(q) \,|\, \epsilon = 1$ or $q \notin \{2,4\}\}$.

ERRATA FOR NUMBER 6

Page 464, Line 16: ~~$\Phi(P)$~~ $\Phi(Z)$