Math 250 B6 Practice Exam 1 6/04/03

You must show your work for full credit. Good luck!

1) Find (a) a row echelon form and (b) the reduced row echelon from for the matrix
$$\begin{bmatrix} 1 & 3 & -1 & 1 & 0 \\ 2 & 6 & -1 & 4 & 15 \\ -1 & -3 & 2 & -1 & 15 \end{bmatrix}.$$
(c) Write the elementary matrices corresponding to the row-operations you performed in part (a).

(d) Find all solutions to the system
$$\begin{array}{rrrrcr} x_1 & +3x_2 & -x_3 & x_4 & = & 0 \\ 2x_1 & +6x_2 & -x_3 & +4x_4 & = & 15 \\ -x_1 & -3x_2 & +2x_3 & -x_4 & = & 15 \end{array}.$$

2. True or False? Justify your answer.

(a) If A and B are matrices such that $AB = I_n$ then A and B are invertible.

(b) If the number of columns of A is greater than the number of rows, then $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions.

(c) The nullity of a an augmented matrix is the number of free variables in a corresponding system of linear equations.

(d) If A is invertible, then A^T is invertible.

(e) If A is invertible, then a reduced row echelon form of A is invertible.

(f) If the number of columns of A is equal to the number of rows, then $A\mathbf{x} = \mathbf{b}$ always has a unique solution.

(g) The identity matrix I_n is invertible.

(h) If A and B are symmetric matrices, then $A + B$ is symmetric.

(i) If $\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_k}$ are vectors in $\mathcal{R}^m$, and $k \geq m$, then these vectors span $\mathcal{R}^m$.

(j) If A is invertible, then A is a product of elementary matrices.

(k) A homogeneous equation is always consistent.

(l) Every finite subset of $\mathcal{R}^m$ is contained in its span.

(m) A set of vectors in $\mathcal{R}^n$ is linearly independent if and only if none of the vectors is a scalar multiple of another.

3. Compute (a) A^2, (b) A^{-1}, and (c) A^T for $A = \begin{bmatrix} 0 & 2 & 1 \\ 0 & 1 & 2 \\ 3 & 0 & 0 \end{bmatrix}$. Check your answer by multiplying A by A^{-1}.

4. Find a set of vectors as small as possible that has the same span as
$$\left\{ \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

5. Prove the following:

(a) If the equation $A\mathbf{x} = \mathbf{b}$ has two solutions, then it has infinitely many solutions.

(b) If $\mathbf{v_1}$ and $\mathbf{v_2}$ are linearly independent, then $\mathbf{v_1}$ and $\mathbf{v_1} + \mathbf{v_2}$ are linearly independent.

(c) If $\mathbf{v_1}$ and $\mathbf{v_2}$ span a set V, then $\mathbf{v_1}$ and $\mathbf{v_1} + \mathbf{v_2}$ also span V

(d) If $\mathbf{v_1}$ and $\mathbf{v_2}$ are solutions to $A\mathbf{x} = \mathbf{0}$ then $\mathbf{v_1} + \mathbf{v_2}$ is a solution to $A\mathbf{x} = \mathbf{0}$.

(e) If A is invertible, then A^2 is also invertible.