

DIFFERENTIATION

Definition:

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

General formulas:

Product: $(uv)' = u'v + uv'$

Quotient: $(u/v)' = [u'v - uv']/v^2$

Chain rule: $[f(u)]' = f'(u) \cdot u'$

Constant multiple: $(cu)' = cu'$

Inverse: $dx/dy = 1/(dy/dx)$

Special functions:

Constants: $c' = 0$

Powers: $\frac{d}{dx} x^n = nx^{n-1}$

Exponential, logarithmic:

$$\frac{d}{dx} e^x = e^x; \quad \frac{d}{dx} \ln(x) = 1/x$$

$$\frac{d}{dx} a^x = \ln a \cdot a^x$$

Trigonometric:

$$\frac{d}{dx} \sin(x) = \cos(x); \quad \frac{d}{dx} \cos(x) = -\sin(x)$$

$$\frac{d}{dx} \sec(x) = \sec(x) \tan(x); \quad \frac{d}{dx} \csc(x) = -\csc(x) \cot(x)$$

$$\frac{d}{dx} \tan(x) = \sec^2(x); \quad \frac{d}{dx} \cot(x) = -\csc^2(x)$$

$$\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}; \quad \frac{d}{dx} \cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}; \quad \frac{d}{dx} \cot^{-1}(x) = \frac{-1}{1+x^2}$$

$$\frac{d}{dx} \sec^{-1}(x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx} \csc^{-1}(x) = \frac{-1}{x\sqrt{x^2-1}}$$

Mean Value Theorem:

$f(x)$ cont. on $[a, b]$, diff. on (a, b) : one can solve

$$f'(x) = \frac{f(b) - f(a)}{b - a},$$

with $a < x < b$.

GRAPHING

Symmetry:

Even: $f(-x) = f(x)$ (symmetric)

Odd: $f(-x) = -f(x)$ (skew symmetric)

Asymptotes:

Horizontal: $\lim_{x \rightarrow \pm\infty} f(x) = a$ (two cases)

Vertical: $\lim_{x \rightarrow a^\pm} f(x) = \pm\infty$ (four cases)

Increasing/Decreasing:

f' Positive on interval: Increasing

f' Negative on interval: Decreasing

Local maxima/minima:

Critical numbers: $f'(x) = 0$, or undefined;

or *Endpoints*

Concavity, inflection:

$f''(x) > 0$: upward; $f''(x) < 0$: downward

$f''(x)$ changes sign: Inflection (\sim)

DIFFERENTIALS AND NEWTON'S METHOD

$$dy = y'dx; \quad y \approx y_0 + dy$$

$$f(a + \Delta x) \approx f(a) + f'(a)\Delta x$$

$$\text{Newton's method: } x^{\text{new}} = x - f(x)/f'(x)$$

(iterate)

Intermediate Value Theorem:

If $f(x)$ is continuous on $[a, b]$, $f(a) < N < f(b)$, then the equation $f(x) = N$ is solvable, with $a < x < b$.

LIMITS

$$\text{L'Hospital: } \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

– (when applicable)

$$\lim_{n \rightarrow \infty} (1 + 1/n)^n = \lim_{h \rightarrow 0} (1 + h)^{1/h} = e$$

Squeeze Theorem:

If $g_1(x) \leq f(x) \leq g_2(x)$ near a ,

and $\lim_{x \rightarrow a} g_1(x) = \lim_{x \rightarrow a} g_2(x) = L$,

then $\lim_{x \rightarrow a} f(x) = L$.

$$\text{Example: } \lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$$

INTERPRETATIONS OF DERIVATIVES

1st: Velocity or rate of change

2nd: Acceleration

Logarithmic: Relative rate of change

Slope of tangent line

INTEGRATION

Integration gives the *signed area* between the curve and the x -axis (above–below).

Fundamental Theorem of Calculus:

(f continuous:) $\int_a^b f(x)dx$ is $F(b) - F(a)$,

with $F(x)$ an antiderivative.

Formulas:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int \sec(x)dx = \ln |\sec(x) + \tan(x)| + C$$

Read the **differentiation formulas** from right to left!

ALGEBRA

Slope: $\Delta y/\Delta x$; Distance: $\sqrt{(\Delta x)^2 + (\Delta y)^2}$

Quadratic formula: $ax^2 + bx + c = 0$: then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$(a - b)(a + b) = a^2 - b^2$$

$$\ln(a^b) = b \ln(a)$$

$$\ln(ab) = \ln(a) + \ln(b); \quad \ln(1/b) = -\ln(b)$$

$$\ln(1) = 0; \ln(e) = 1$$

$$\log_a(x) = \frac{\ln(x)}{\ln(a)}$$

Area:

Triangle: $1/2$ base \times altitude

Circle: πr^2 ; Sphere (surface): $4\pi r^2$

Volume:

Box: Product of dimensions.

Sphere (inside): $\frac{4}{3}\pi r^3$

Cylinder: Base area \times Height

Cone: $\frac{1}{3}$ Base area \times Height

TRIGONOMETRY

Values:

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

Right Triangles:

sine: opposite/hypotenuse

cosine: adjacent/hypotenuse

tangent: opposite/adjacent

secant: $1/\text{cosine}$

Multiples:

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\sin^2(x/2) = \frac{1 - \cos(x)}{2}; \quad \cos^2(x/2) = \frac{1 + \cos(x)}{2}$$

More identities:

$$\sin(x \pm y) = \sin(x) \cos(y) \pm \cos(x) \sin(y)$$

$$\cos(x \pm y) = \cos(x) \cos(y) \mp \sin(x) \sin(y)$$

$$\tan(x + y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x) \tan(y)}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

Co-functions:

$$\cos(x) = \sin(\pi/2 - x); \quad \cot(x) = \tan(\pi/2 - x)$$

$$\csc(x) = \sec(\pi/2 - x)$$

NUMBERS (*rough approximations*)

$$\pi \approx 3.14 \quad e \approx 2.7 \quad \sqrt{2} \approx 1.4 \quad \sqrt{3} \approx 1.7$$

$$\ln 2 \approx .7 \quad \ln 10 \approx 2.3$$