Calculus 151 Problems, Week 7

1. Given that f and g are differentiable functions, and that $h(z) = f(g(z))$, fill in the gaps in the following table:

a	$g(a)$	$g'(a)$	$f(a)$	$f'(a)$	$h(a)$	$h'(a)$
0	1	-3	3	-2	$-1/3$	...
1	2	0	...	-7	...	...
2	2	-1	-1	...	...	3

2. Let f be a function such that $f(1) = 2$ and $f'(x) = \sqrt{x^3 + 1}$ (there is no simple formula for $f(x)$ itself).

(a) Find the linear approximation to $f(x)$ near $a = 1$, and use it to estimate $f(1.1)$.

(b) Find the quadratic approximation to $f(1.1)$ near $a = 1$, and use it to estimate $f(1.1)$.

3. Graph the function $f(x) = x^2 - 1$ in the range $-3 \leq x \leq 3$.

(a) If Newton's method is applied repeatedly with initial value $x_0 = 2$, a sequence of numbers $\{x_n\}$ is obtained (here x_n is the result of applying Newton's method to x_{n-1}). Display on your graph how x_1 is obtained from the tangent line at $(x_0, f(x_0))$, and then repeat the tangent line construction to obtain x_2, and x_3. What do you think will happen to x_n as $n \to \infty$?

b) Obtain a formula for x_n in terms of x_{n-1}. Taking $x_0 = 2$, calculate x_n for $n = 1, 2, 3, 4$.

(c) What value of n would you need to take to get x_n to be equal to the root $x = 1$ of the equation $f(x) = 0$ within an error of 10^{-12}. Try to guess a value n from the evidence in (b), and then check your guess by calculation.

4. (a) Graph the curves $y_1 = \sin x$ and $y_2 = 4\cos x$. Use the cursor and the ZOOM BOX to find the the point of intersection of the curves that has the smallest positive x-value. Read off the approximate coordinates (x_0, y_0) of this point.

(b) Write out the Newton's method iteration to find the solutions to the equation $\sin x - 4\cos x = 0$. Give a formula for x_{n+1} in terms of x_n for any n.

(c) Starting with the value x_0, calculate x_1 and x_2 using your formula in (b). This is easy if you use the SEQUENCE mode in your calculator (see the manual). Compare your results with the approximate values that you can get for the solution by ZOOMing in on the point of intersection using the calculator.

5. Graph the function $Q(x) = x^5 + x^3 + x$ on the range $0 < x < 2$ to see that it is a one-to-one function with range $0 < y < 42$. Suppose that R is the function inverse to Q. Compute $R(3)$, $R'(3)$ and $R''(3)$. (Use implicit differentiation. Don't try to find a formula for R; there isn't any simple one.)