Math 354 Spring 2005

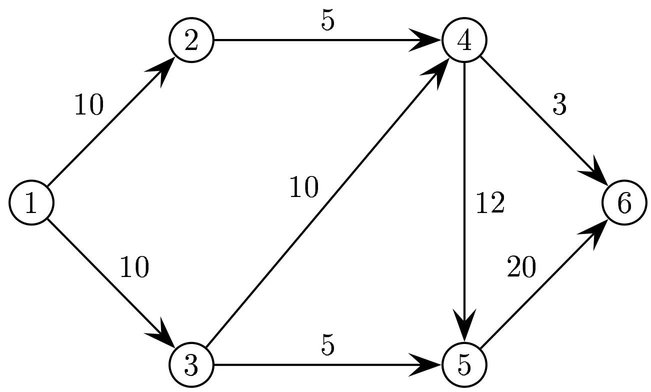
Final Exam Review

Most importantly: Do Problems #1 - #5. Some version of each one of those problems will probably show up on the final.

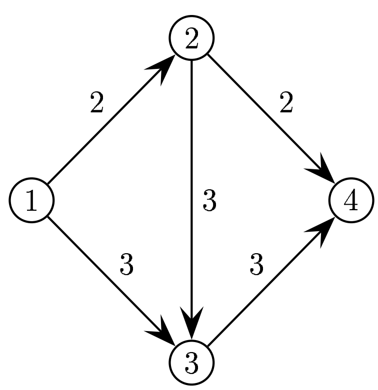
Then: Review the first two midterms. Several questions on the final will be like problems from those.

But also: Here are (#6-8) three problems that are a little different from problems that have been on the exams. Problems like these could also appear on the final.

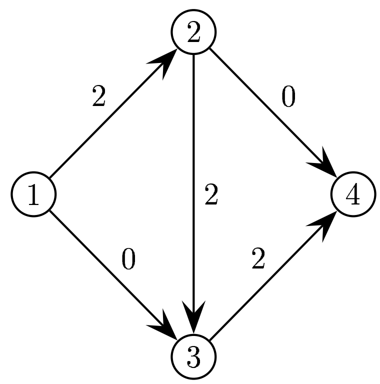
1 Find a maximal flow from the source (vertex 1) to the sink (vertex 6) in the following network using the Ford-Fulkerson Algorithm. Prove that your flow is optimal by exhibiting a cut with the same capacity. (Extra copies of the graph are included at the end of this review for you to use.)



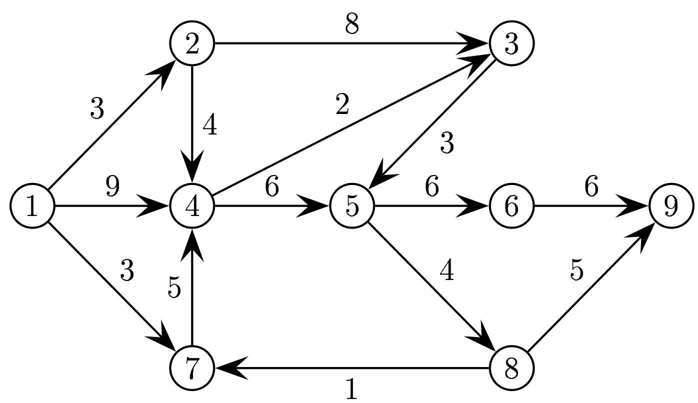
2 Consider the following network with source 1 and sink 4.



Extend the following flow to an optimal flow using the Ford-Fulkerson Algorithm. Then prove that you have achieved an optimal flow by exhibiting a cut with the same capacity. (There are also extra copies of this graph at the end of the review.)



3 Use the algorithm from Section 5.5 of your book (which is also the algorithm I presented in class) to compute the shortest path between vertex 1 and vertex 9 in the following digraph.



4 Use the method we have learned for the assignment problem to solve the following integer programming problem. Be careful to note that this is a maximization problem.

The Rutgers football program needs to assign positions to four new players. Each can play quarterback (QB), tight-end (TE), wide receiver (WR), and cornerback (CB) moderately well, but the team can have only one new player at each of these positions. The chart below shows the value that each player will add to each position.

	QB	TE	WR	CB
Ben	5	8	4	4
Bob	3	3	4	3
Jim	4	5	3	5
Ross	3	3	5	6

How should the team assign positions to these four players in order to maximize the total value added?

5 Use the Transportation Algorithm to solve the following integer programming problem.

A company has two factories and two warehouses. The cost to transport 1 unit of material between the factories and warehouses appears below.

	to	
from	1	2
1	5	7
2	5	6

Both factories can supply 100 units, and both warehouses demand 75 units. Minimize the total transportation cost.

6 Consider the following linear programming problem.

$$\begin{array}{l}\text{Maximize } z = 4x_1 + 6x_2 + 2x_3 \\ \text{subject to} \\ \begin{array}{ccccccc} x_1 & + & x_2 & + & x_3 & \leq & 10 \\ x_1 & + & 4x_2 & & & \leq & 15 \\ x_1 & & & + & x_3 & \leq & 6 \\ x_1, & & x_2, & & x_3 & \geq & 0 \end{array}\end{array}$$

Applying the Simplex Method to this problem yields the following final tableau

		4	6	2	0	0	0	
$\underline{c}_B$		x_1	x_2	x_3	u_1	u_2	u_3	
0	u_1	0	0	$\frac{1}{4}$	1	$-\frac{1}{4}$	$-\frac{3}{4}$	$\frac{7}{4}$
6	x_2	0	1	$-\frac{1}{4}$	0	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{9}{4}$
4	x_1	1	0	1	0	0	1	6
		0	0	$\frac{1}{2}$	0	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{75}{2}$

This tableau represents the optimal solution

$$\underline{x} = \begin{bmatrix} 6 \\ \frac{9}{4} \\ 0 \end{bmatrix}$$

Suppose now that the problem is changed to "Maximize $z = (4+\Delta)x_1 + 6x_2 + 2x_3$" for some real number Δ. For what range of Δ is

$$\underline{x} = \begin{bmatrix} 6 \\ \frac{9}{4} \\ 0 \end{bmatrix}$$

still an optimal solution for the new problem? (Hint: you may want to try a specific value or two for Δ first, to get a feel for what I'm asking.)

7 Given that

$$\underline{x} = \begin{bmatrix} 3 \\ 1 \\ 6 \end{bmatrix}$$

is a solution to the primal problem

$$\begin{array}{l} \text{Maximize } z = x_1 + 4x_2 + 5x_3 \\ \text{subject to} \\ \begin{array}{rcrcrcl} -x_1 & + & x_2 & + & x_3 & \leq & 4 \\ 3x_1 & + & x_2 & + & x_3 & \leq & 16 \\ & & x_2 & & & \geq & 1 \\ x_1, & & x_2, & & x_3 & \geq & 0 \end{array} \end{array}$$

use the Principle of Complementary Slackness to find a solution to the dual problem

$$\begin{array}{l} \text{Minimize } z = 4w_1 + 16w_2 - w_3 \\ \text{subject to} \\ \begin{array}{rcrcrcl} -w_1 & + & 3w_2 & & & \geq & 1 \\ w_1 & + & w_2 & - & w_3 & \geq & 4 \\ w_1 & + & w_2 & & & \geq & 5 \\ w_1, & & w_2, & & w_3 & \geq & 0 \end{array} \end{array}$$

8 Consider the following linear programming problem.

$$\begin{array}{l} \text{Maximize } z = 3x_1 - x_2 + 2x_3 + 4x_4 \\ \text{subject to} \\ \begin{array}{rcrcrcrcl} & & x_2 & + & 7x_3 & + & 2x_4 & \geq & 3, \\ x_1 & + & 2x_2 & + & x_3 & & & = & 9, \\ 2x_1 & + & 3x_2 & + & x_3 & - & 4x_4 & \leq & 7, \end{array} \\ x_j \geq 0, \;\; j = 1, 2, 3, 4. \end{array}$$

When solving it, the following tableau came up

	x_1	x_2	x_3	x_4	u_1	u_2	
x_4	0	$\frac{1}{4}$	$\frac{1}{4}$	1	0	$-\frac{1}{4}$	$\frac{11}{4}$
u_1	0	$-\frac{1}{2}$	$-\frac{13}{2}$	0	1	$-\frac{1}{2}$	$\frac{5}{2}$
x_1	1	2	1	0	0	0	9
	0	8	2	0	0	16	38

Could it be correct? Explain.

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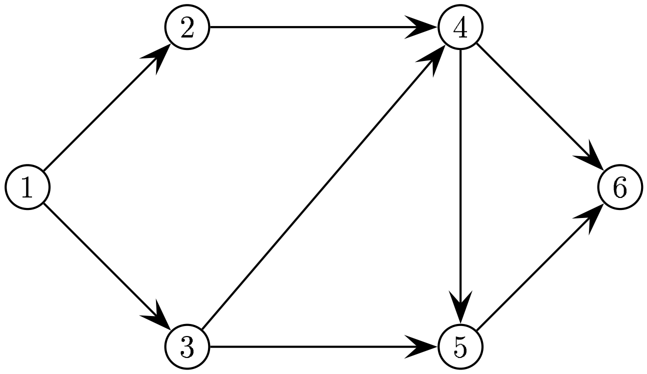

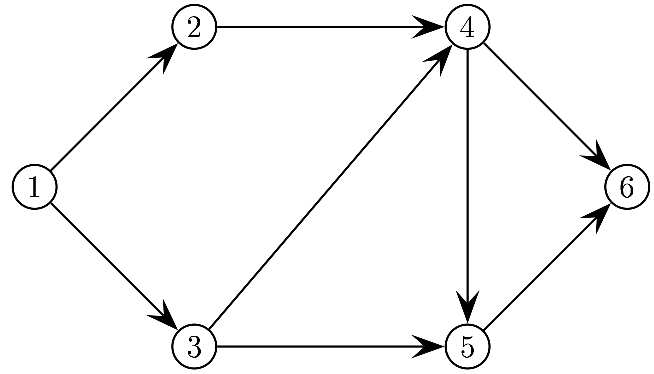

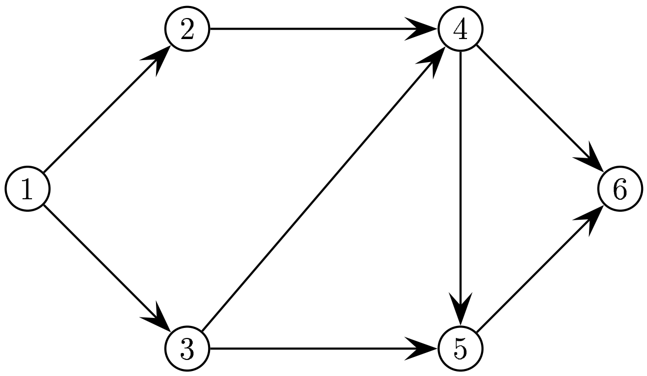

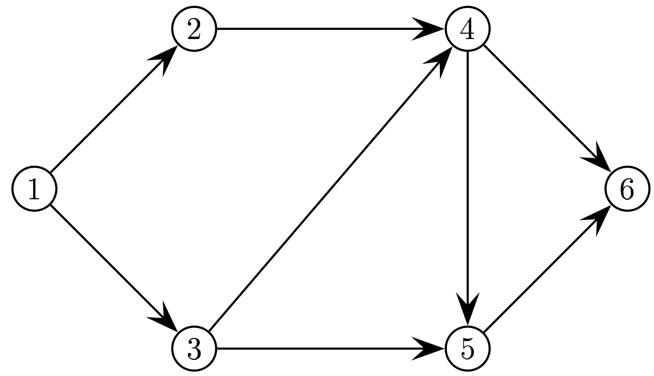

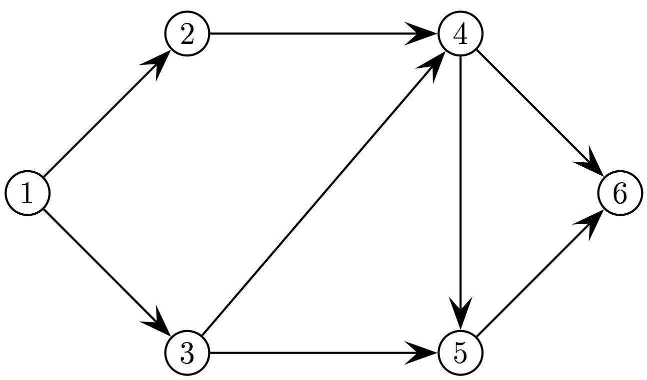

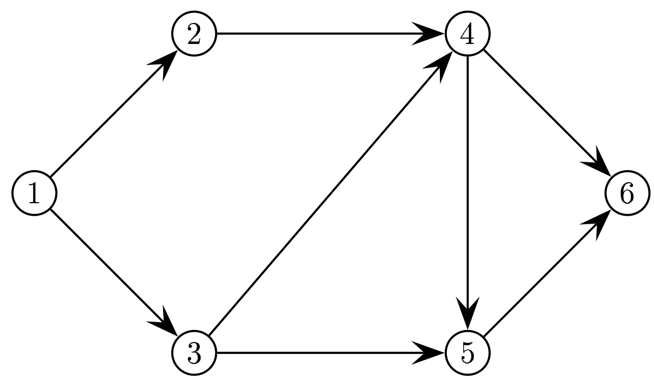

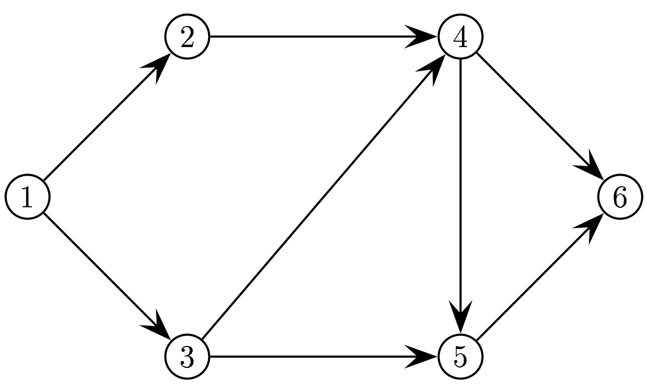

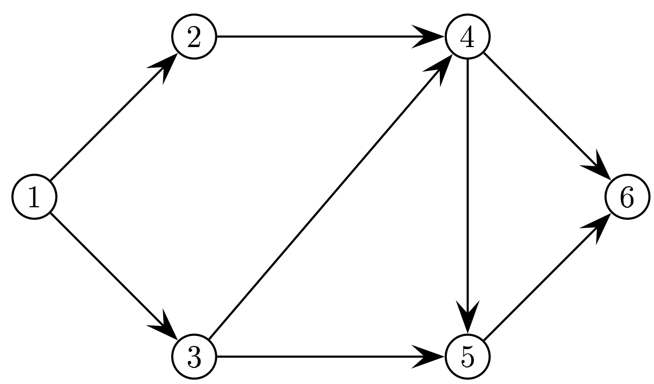

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