Math 354 Spring 2005

Review problems for Midterm #1

First: Midterm #1 covers Chapter 1 and 2. In particular, this means that it does not explicitly cover linear algebra. Also, I promise there will not be any proofs.

1 Consider the following scenario. Model it as a linear programming problem. Be sure to state explicitly what each of your decision variables $x_1, x_2, \ldots$ represent. *Do not attempt to solve the LPP.*

A cattle rancher uses three types of cattle feed: type 1, type 2, and type 3. Type 1 costs \$1.50 per pound, type 2 costs \$3.50 per pound, and type 3 costs \$2.00 per pound. The rancher wants to meet the following minimum daily requirements for each animal. Each day, each animal should have at least 120 mg of vitamin A, 180 mg of vitamin B, and 100 mg of vitamin C.

The following chart shows the number of mg per pound of each vitamin in the three types of feed:

Vitamin	Type 1	Type 2	Type 3
A	8	2	20
B	9	11	5
C	1	10	20

Because of protein content, however, an animal cannot eat more than 15 pounds of type 1, 10 pounds of type 2, and 5 pounds of type 3 cattle feed per day. How many pounds of each type of cattle feed should the rancher purchase per day in order to minimize the cost, while still meeting the nutritional minimum daily requirements?

2 Convert the following LPP into (a) standard form, and (b) canonical form:

$$\text{minimize } z = x_1 - 4x_2 + 5x_3$$

subject to

$$\begin{array}{rrrcl} x_1 & & +x_3 & \leq & 5 \\ & x_2 & +3x_3 & \geq & 7 \\ 2x_1 & +9x_2 & & = & 11 \\ & & & & \\ x_1, & x_2 & & \geq & 0 \\ & & x_3 & & \text{unconstrained} \end{array}$$

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3 Consider the LPP

$$\text{maximize } z = 2x_1 + 4x_2$$

subject to

$$\begin{array}{rrrcl} 5x_1 & +3x^2 & +5x_3 & \leq & 15 \\ 10x_1 & +8x_2 & +15x_3 & \leq & 40 \\ \\ x_1, & x_2 & x_3 & \geq & 0 \end{array}$$

First, sketch the set of feasible solutions. Then find the extreme points of this set. Then find the optimal solution(s).

4 Find an optimal solution to the following LPP using the simplex method.

$$\text{maximize } z = x_1 + 3x_2 + 5x_3$$

subject to

$$\begin{array}{rrrcl} 2x_1 & -5x_2 & +x_3 & \leq & 3 \\ x_1 & +4x_2 & & \leq & 5 \\ \\ x_1, & x_2, & x_3 & \geq & 0 \end{array}$$

5 Find an optimal solution to the following LPP using the two-phase simplex method.

$$\text{maximize } z = -x_1 - 2x_2$$

subject to

$$\begin{array}{rrrrcr} x_1 & +2x_2 & -x_3 & & = & 3 \\ 3x_1 & +4x_2 & & -x_4 & = & 10 \\ \\ x_1, & x_2, & x_3, & x_4 & \geq & 0 \end{array}$$