Rigorous verification of the hyperbolicity of dynamical systems and its applications

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In this talk, we propose a rigorous computational algorithm for proving the uniform hyperbolicity of dynamical systems. Given a chain recurrent invariant set, we first prove the quasi-hyperbolicity, a weaker notion of hyperbolicity, and then use the chain-recurrence condition to conclude the uniform hyperbolicity. This enables us to avoid the difficulty of constructing a hyperbolic splitting and a adapted metric. The method involves interval arithmetic, the subdivision algorithm and some graph theoretical algorithms.

As an application of the algorithm, we prove the existence of non-trivial loops in the hyperbolic horseshoe parameter set of the complex Hénon map. This implies that the topology of the 2-dimensional generalization of the Mandelbrot set is totally different from that of the original one. Furthermore, we show that the monodromy of loops satisfying a certain symmetry condition determines the dynamics of the real Hénon map whose parameter is on the loop. Using this relation between real and complex Hénon map, we obtain the complete description of the dynamics of real Hénon map for some parameter regions.