Automatic generation of Taylor integrators for high order variational equations of ODEs

Àngel Jorba

Departament de Matemàtica Aplicada i Anàlisi
Universitat de Barcelona,
Gran Via 585, 08007 Barcelona, Spain
E-mail: <angel@maia.ub.es>

In a previous work (Jorba & Zou, 2005) we presented a software that, given a function f (belonging to a suitable class), generates code to compute the jet of derivatives of the solution of $x' = f(t, x)$ at a given point (t_m, x_m). The order of the jet is given at run time. The software also generates code to compute, adaptively, an order and step size and to add the resulting Taylor series to predict a new point on the solution. Therefore, the output is a complete time-stepper with automatic order and step size control. The generated code is ANSI C, and it is possible to use different floating point arithmetics, like GMP or MPFR. This software is freely downloadable from `http:/www.maia.ub.es/~angel/taylor/` or from `ftp://ftp.ma.utexas.edu/pub/mzou`.

One of the improvements for the package is the automatic computation of variational equations of arbitrary order. To this end, we will use the ideas of automatic differentiation to compute the derivatives (up to a prescribed order) of the final point of the trajectory w.r.t. the initial condition. For a concrete application of these ideas, see Makino & Berz, 2006. In particular, to apply these techniques one requires a fast algebraic manipulator for polynomials of several variables. In the presentation we will discuss the implementation (in C/C++) of this algebraic manipulator, and we will show how it is used to compute high order variationals. The examples will come from Celestial Mechanics.