Global bifurcation and dynamics of localized moving patterns in heterogeneous media

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Heterogeneity in the media is the most important and ubiquitous type of external perturbation in dissipative systems. We study how heterogeneity influences the propagating manner of spatially localized traveling waves, especially when they encounter heterogeneities of a bump or periodic type. A variety of outputs emerge through the interaction between the traveling patterns and the heterogeneity-induced-ordered-pattern (HIOP), including pinning, splitting, rebound, and penetration. One of the origins of such rich behaviors is that the pulse has potential instabilities that display a variety of dynamics such as drift, saddle-node, and Hopf bifurcations. A reduction from a partial differential equation (PDE) to a finite-dimensional ordinary differential equation (ODE) is possible near such singularities of codim1 and codim2 types, and the resulting system inherits most of the essential dynamics from the original PDE. It turns out that there are hidden HIOPs associated with the critical points of reduced ODEs. They play a pivotal role in understand the transitions among different responses; in fact, various bifurcations such as Hopf and heteroclinic ones of HIOPs cause the onset of those transitions. For a periodic type of heterogeneity, spatio-temporal chaos is also observed numerically for the motion of a pulse as a result of the interplay between the internal driving force and the potential landscape created by heterogeneity.