Chaotic braided solutions in the Swift-Hohenberg equation

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The Swift-Hohenberg equation is a fourth order parabolic PDE that models certain aspects of pattern formation, such as the finite wavelength instability in Rayleigh-Benard convection. We focus on stationary solutions of the equation in one dimension, and interpret solutions of this ODE as braided strands. Using a variational principle this leads to topological forcing results. On the other hand, recent advances in rigorous numerics allow us to prove the existence of a certain periodic solution with geometric properties that, via the forcing results, imply chaotic dynamics for the Swift-Hohenberg ODE for a large range of parameter values.