Topology-Guided Sampling of Complicated Random Patterns

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Many stochastic partial differential equation models arising in applications generate complex time-evolving patterns which are hard to quantify due to the lack of any underlying regular structure. The influence of stochasticity leads to variations in the detail structure of the patterns and forces one to concentrate on rougher common geometric features. In many of these instances, such as for example in phase-field type models in materials science, one is interested in the geometry of sublevel sets of a function in terms of their topology, in particular, their homology.

Recent computational advances make it possible to compute the homology of discrete structures efficiently and fast. Such methods can be applied to the above situation if the sublevel sets of interest are approximated using an underlying discretization of the considered evolution equation. Yet, this method immediately raises the question of the accuracy of the computed homology. In this talk, I will present a probabilistic approach which gives insight into the suitability of the above method in the context of random fields. We will obtain explicit probability estimates for the correctness of the homology computations, which in turn yield a-priori bounds for the suitability of certain grid sizes as well as information on the optimal location of sampling points.