Rigorous verification of cocoon bifurcations in the Michelson system.

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We present [KWZ] a proof of the existence of *cocoon* bifurcations for the Michelson system

$$\dot{x} = y, \quad \dot{y} = z, \quad \dot{z} = c^2 - y - \frac{1}{2}x^2,$$

where $(x, y, z) \in \mathbb{R}^3$ and $c \in \mathbb{R}_+$ is a parameter, based on the theory given in [DIK]. The main difficulty lies in the verification of the (topological) transversality of the unstable set W^u of a nonyperbolic fixed point P for suitable Poincaré map and the stable manifold W^s of an equilibrium. Rigorous integration of second order variational equations [WZ] is used to prove the existence of a Lyapunov function in a neighborhood of P which gives an estimation and some monotonicity properties of W^u. Then, the existence of topologically transverse intersection of W^u and W^s is proven by means of covering relations and cone conditions tools.

These new techniques developed in this paper will have broader applicability to similar global bifurcation problems.

[DIK] F. Dumortier, S. Ibáñez, and H. Kokubu, *Cocoon bifurcation in three dimensional reversible vector fields*, Nonlinearity **19** (2006), 305–328.

[KWZ] H. Kokubu, D. Wilczak, P. Zgliczyński, *Rigorous verification of cocoon bifurcations in the Michelson system*, Nonlinearity, **20** (2007), 2147–2174.

[WZ] D. Wilczak, P. Zgliczyński, *C^r-Lohner algorithm*, in review.